# Banking Panics and Policy Responses

Huberto M. Ennis
Research Department, Federal Reserve Bank of Richmond
*Huberto.Ennis@rich.frb.org*

Todd Keister
Research and Statistics Group, Federal Reserve Bank of New York
*Todd.Keister@ny.frb.org*

December 17, 2009

**Abstract**

We study how banking panics unfold in a version of the Diamond and Dybvig (1983) model with limited commitment. In our model, a policy maker with full commitment power could costlessly eliminate the possibility of a run on the banking system. When the policy maker's ability to commit is limited, however, self-fulfilling runs easily arise. We construct equilibria in which depositors run on the banking system with positive probability and we show that a bank run in this setting is necessarily partial, with only some depositors participating. We also show that a run naturally occurs in waves, with each wave of withdrawals prompting a further policy response from the banking authority. In this way, the interplay between the actions of depositors and the responses of the policy maker shapes the course of the crisis.

---

An earlier version of this paper circulated under the title "Commitment and Equilibrium Bank Runs." We thank seminar participants at the Autònoma de Barcelona, Carlos III, Delaware, Kansas, Kentucky, Michigan State, Penn State, Pompeu Fabra, Rice, Southampton, Toulouse, University at Albany (SUNY), the Federal Reserve Banks of Chicago, Cleveland and Philadelphia, the Latin American Meetings of the Econometric Society, the Midwest Macroeconomics Meetings, the SAET meetings, the SED meetings, and the conference on Money, Banking, and Asset Markets at the University of Wisconsin. We are particularly grateful to Francesca Carapella, Ed Green, Bob King, Guido Lorenzoni, Karl Shell, Harald Uhlig, Ludo Visschers, and Neil Wallace for useful comments. The views expressed herein are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York, the Federal Reserve Bank of Richmond, or the Federal Reserve System.

# 1 Introduction

Recent events have renewed interest in studying how policy makers respond to banking panics and related events. Several episodes during the current financial crisis have been compared to "old fashioned" banking panics; examples include the collapse of the market for asset-backed commercial paper in 2007, the near-failure of the investment bank Bear Stearns in March 2008, and the surge of withdrawals from money market mutual funds in September 2008, to name only a few. Each of these events led to a reaction by policy makers in central banks and in government. Moreover, traditional bank runs – where retail depositors rush to withdraw from their local banks – remain a major issue in some economies, as demonstrated by events in Argentina (in 2001), Russia (in 2004) and elsewhere. Policy makers respond to these events as well, often by freezing deposits and/or rescheduling the liabilities of the banking sector.

Investors anticipate that policy makers will respond to a crisis, of course, and the anticipated response influences their behavior. We study the interplay between investor's decisions and the responses of policy makers in a version of the Diamond and Dybvig (1983) model of bank runs with limited commitment. The assumption of limited commitment seems particularly appropriate for studying bank runs and other crisis; it amounts to assuming that policy makers cannot credibly promise to refrain from intervening when an (ex post) improvement in resource allocation is possible. We use the model to examine the essential trade-offs facing policy makers during a banking panic and we derive the time-consistent banking policy in equilibrium. We show that a lack of policy commitment can play an essential role in both allowing self-fulfilling banking panics to arise and in determining the pattern that such panics follow.

The previous literature has assumed, often implicitly, that policy makers can commit to follow a specific course of action in the event of a crisis. To see why the issue of commitment is important, consider the standard version of the Diamond-Dybvig model with no aggregate uncertainty. Individual agents are unsure about when they will need to consume and, therefore, pool their resources in a bank for insurance purposes. In an environment with commitment, a benevolent banking authority sets a payment schedule – a complete specification of how much each depositor who withdraws early will receive – before depositors make their withdrawal decisions. By threatening to freeze all remaining deposits if too many depositors withdraw early, this authority can guarantee the solvency of the banking system. When solvency is guaranteed, it is a dominant strategy for each depositor to wait to withdraw unless she truly needs to consume early. Hence, commitment to an appropriate course of action can rule out the possibility of a banking panic and ensure the efficient outcome.[1]

We study this same model, but in an environment where the banking authority cannot pre-commit to a course of action. Instead, it will respond optimally to whatever situation arises. When faced with a run in this environment, the banking authority will not choose to freeze all remaining deposits, because doing so would deny consumption to some agents who have a true need to consume early.[2] The optimal response is to allow additional withdrawals, but at a discount to their face value. The appropriate discount depends on how the banking authority expects the remaining depositors to behave. The behavior of these remaining depositors, in turn, is influenced by the level of the discount imposed by the banking authority. The banking authority and all depositors fully anticipate and optimally react to each others' behavior in our model. The equilibrium pattern of withdrawals and discounts is thus determined by the interplay between depositors' withdrawal decisions and the responses of the banking authority.

We show that when depositors are sufficiently risk averse, there exists an equilibrium of the model in which a bank run occurs with positive probability. Despite the simplicity of the environment, the structure of the equilibrium we construct is surprisingly rich. The initial run is necessarily partial, with only some depositors participating. Once the number of early withdrawals passes a certain threshold, the banking authority realizes that a run is underway and imposes a discount on all further early withdrawals. The run may halt at this point or it may continue, leading the banking authority to announce another, more severe discount on withdrawals. A bank run thus occurs in "waves," with each wave of withdrawals prompting a further reaction by the banking authority. The number of waves that occur in equilibrium is stochastic and can be arbitrarily large.

This dynamic "wave" structure is fundamentally different from the type of bank run studied in the previous literature, where depositors run either *en masse* or not at all. After the first wave

1 In a related model, de Nicolò (1996) shows how run equilibria can be ruled out under commitment without freezing deposits by using a priority-of-claims provision on final date resources. Deposit freezes (sometimes called *suspensions of convertibility*) have been studied in similar settings by Gorton (1985), Chari and Jagannathan (1988), and Engineer (1989).

2 In earlier work (Ennis and Keister, 2009a), we showed that a full deposit freeze is not *ex post* efficient in the event of a run. We also discussed institutional features that often shape a government's response to a run, with a focus on events in Argentina in 2001-2 and other recent banking crises.

of early withdrawals, the banking authority in our model is able to infer that a partial run has taken place, but it does not know whether the run will continue. The structure of the equilibrium is such that, at each decision point, the banking authority is optimistic that the run has ended. This optimism leads it to offer a relatively high degree of risk sharing to the remaining depositors, which, in turn, leaves the banking system susceptible to a continued run. In this way, our model suggests that the combination of a lack of commitment together with optimism on the part of policy makers during a crisis may lie at the root of the problem of self-fulfilling runs. We believe this is a new and potentially important insight into the fundamental causes of financial fragility.

Our analysis contributes to a small but growing literature on discretionary policy and multiple equilibria. Most of the work on issues related to time inconsistency has studied situations where the inability of a policy maker to commit leads to an inefficient outcome in the unique equilibrium. In our setting, the efficient outcome is always *an* equilibrium. A policy maker with commitment power can rule out other (*i.e.*, bank run) equilibria, but a lack of commitment power allows such equilibria to arise. Hence, our analysis is more in line with the flood control example in Kydland and Prescott (1977). In that example, a commitment to not invest in flood control would convince private agents to not build on a flood plain. However, if the policy maker cannot commit, there is an equilibrium in which agents build on the flood plain and, as a result, the policy maker ends up investing in flood control.[3] This second type of inefficiency resulting from a lack of commitment power has been studied in the context of fiscal policy by Glomm and Ravikumar (1995) and in the context of monetary policy by Albanesi, et al. (2003) and King and Wolman (2004). Our analysis shows how these same forces naturally generate self-fulfilling bank run equilibria in the well-known Diamond-Dybvig framework and that these equilibria have a rich dynamic structure.

The rest of the paper is organized as follows. In the next section, we describe the environment and the definition of equilibrium in both the commitment and the no-commitment case. We briefly analyze equilibrium in the commitment case in Section 3, showing that bank runs never occur and the first-best allocation always obtains. Section 4 contains the main result: a construction of bank run equilibria in the no-commitment case, while Section 5 illustrates the properties of these equilibria. We offer some concluding remarks in Section 6.

[3] See King (2006) for a more formal analysis of this problem.

# 2 The Model

We work with a fairly standard version of the Diamond-Dybvig model that includes an explicit sequential service constraint. We begin by describing the physical environment and the first-best allocation of resources in this environment.

## 2.1 The environment

There are three time periods: $t = 0, 1, 2$. There is a continuum of agents, whom we refer to as *depositors*, indexed by $i \in [0, 1]$. Each depositor has preferences given by

$$u(c_1, c_2; \theta_i) = \frac{(c_1 + \theta_i c_2)^{1-\gamma}}{1-\gamma},$$

where $c_t$ is consumption in period $t$ and $\theta_i$ is a binomial random variable with support $\Theta = \{0, 1\}$. As in Diamond and Dybvig (1983), we assume that the coefficient of relative risk aversion $\gamma$ is greater than $1$. If the realized value of $\theta_i$ is zero, depositor $i$ is *impatient* and only cares about consumption in period $1$. A depositor's type $\theta_i$ is revealed to her in period $1$ and is private information. Let $\pi$ denote the probability with which each individual depositor will be impatient. By a law of large numbers, $\pi$ is also the fraction of depositors in the population who will be impatient.[4] Note that $\pi$ is non-stochastic; there is no aggregate (*intrinsic*) uncertainty in this model.

The economy is endowed with one unit of the good per capita in period $0$. As in Diamond and Dybvig (1983), there is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the later periods. A unit of the good invested in period $0$ yields $R > 1$ units in period $2$, but only one unit in period $1$.

There is also a banking technology that allows depositors to pool resources and insure against individual liquidity risk. The banking technology is operated in a central location. As in Wallace (1988, 1990), depositors are isolated from each other in periods $1$ and $2$ and no trade can occur among them. However, each depositor has the ability to visit the central location once, either in period $1$ or in period $2$ and, hence, a payment can be made to her from the pooled resources after her type has been realized. We refer to the act of visiting the central location as *withdrawing* from the banking technology.

[4] There are well-known technical issues associated with the formal statement of the law of large numbers in an economy with a continuum of agents. We ignore the technical details here and refer the reader to Al-Najjar (2004) for a discussion, references, and a possible way to deal with such issues.

Depositors' types are revealed in a fixed order determined by the index $i$; depositor $i$ discovers her type before depositor $i'$ if and only if $i < i'$. A depositor knows her own index $i$ and, therefore, knows her position in this ordering.[5] Upon discovering her type, each depositor decides whether or not to visit the central location in period 1. If she does, she must consume immediately; the consumption opportunity in period 1 is short-lived. This implies that the payment a depositor receives from the banking technology cannot depend on any information other than the number of depositors who have withdrawn prior to her arrival. In particular, it cannot depend on the *total* number of depositors who will withdraw in period 1, since this information is not available when individual consumption must take place. This sequential-service constraint follows Wallace (1988, 1990) and captures an essential feature of banking: the banking system pays depositors as they arrive to withdraw and cannot condition current payments to depositors on future information.

Under sequential service, the payments made from the banking technology in period 1 can be summarized by a function $x : [0,1] \rightarrow \mathbb{R}_+$, where the number $x(\mu)$ is the payment given to the $\mu^{\text{th}}$ depositor to withdraw in period 1. Note that the arrival point $\mu$ of a depositor depends not only on her index $i$ but also on the actions of depositors with lower indexes. In particular, $\mu$ will be strictly less than $i$ if some of these depositors choose not to withdraw in period 1. In period 2, we can, without loss of generality, set the payment to each depositor equal to an even share of the matured assets in the banking technology.[6] Therefore, the operation of the banking technology is completely described by the function $x$, which we call the *banking policy*. Feasibility of the banking policy requires that total payments in period 1 not exceed the short-run value of assets, even if all depositors choose to withdraw in that period, that is,

$$\int_0^1 x(\mu)\, d\mu \leq 1. \tag{1}$$

We summarize the behavior of depositor $i$ by a function $y_i : \Theta \rightarrow \{0,1\}$ that assigns a particular action to each possible realization of her type. Here $y_i = 0$ represents withdrawing in period 1 and $y_i = 1$ represents waiting until period 2. We refer to the function $y_i$ as the *withdrawal strategy* of

5 This construction follows Green and Lin (2000) and is a simplified version of that in Green and Lin (2003). None of our results depend on the assumption that depositors know this ordering. The same results would obtain if depositors made their withdrawal decisions before this ordering is realized (as in Diamond and Dybvig 1983, Peck and Shell 2003, and others), but the details would be more complex in some cases.

6 In principle, some type of payment schedule could be applied in period 2 as well. However, since depositors are risk averse and all information about their actions has been revealed at this point, it will always be optimal to divide the assets evenly among the remaining patient depositors. Importantly, the type of priority-of-claims provision studied in de Nicolò (1996) would never be used in a setting without commitment because it is *ex post* inefficient.

depositor $i$, and we use $y$ to denote the *profile of withdrawal strategies* for all depositors.

An *allocation* in this environment consists of an assignment of consumption levels to each depositor in each period. An individual depositor's consumption is completely determined by the banking policy $x,$ the profile of withdrawal strategies $y$, and the realization of her own type $\theta_i$. We can, therefore, define the (indirect) expected utility of depositor $i$ as a function of $x$ and $y$, that is,

$$v_i(x,y) = E\left[u\left(c_{1,i}, c_{2,i}; \theta_i\right)\right],$$

where $E$ represents the expectation over $\theta_i$. Define $U$ to be the integral of all depositors' expected utilities, *i.e.*,

$$U(x,y) = \int_0^1 v_i(x,y)\, di. \tag{2}$$

This expression can be given the following interpretation. Suppose that, at the beginning of period 0, depositors are assigned their index $i$ randomly, with each depositor having an equal chance of occupying each space in the unit interval. Then $U$ measures the expected utility of each depositor before places are assigned. We use $U$ as our measure of aggregate welfare throughout the paper.

## 2.2 The first-best allocation

Consider the problem of a benevolent social planner who can observe depositors' types and can directly control both the banking technology and the withdrawal decisions of depositors. This planner would choose the variables $x$ and $y$ to maximize $U$ subject to the feasibility constraint in (1). In other words, the planner will choose how much and in which period each depositor consumes, contingent on types and subject to the sequential service restriction described above. We call the allocation this planner would generate the (full information) *first best*.

The solution to this problem parallels that in the standard Diamond-Dybvig model. First, note that the planner would direct all impatient depositors to withdraw in period 1 and all patient depositors to withdraw in period 2; that is, the planner would set

$$y_i(\theta_i) = \theta_i \quad \text{for all } i.$$

Furthermore, because depositors are risk averse and there is no aggregate uncertainty, depositors of a given type will all receive the same amount of consumption. Let $c_1$ denote the level of consumption provided to impatient depositors and $c_2$ the level of consumption provided to patient depositors. These numbers will be chosen to solve

$$\max_{\{c_1,c_2\}} \pi \frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(c_2)^{1-\gamma}}{1-\gamma} \tag{3}$$

subject to

$$(1-\pi)c_2 = R(1-\pi c_1).$$

As is well known, the solution will satisfy $c_2^* > c_1^* > 1$. The payment schedule associated with the first-best allocation then sets[7]

$$x(\mu) = c_1^* \text{ for all } \mu \in [0, \pi]. \tag{4}$$

Note that the first-best allocation described here is the same allocation the planner would choose in an environment without the sequential service constraint, where it could first observe all depositor's types and then assign a consumption allocation. In our setting, where there is no aggregate uncertainty, the sequential service constraint is non-binding in the planner's problem. However, as we discuss below, the constraint is an important restriction in the decentralized economy where types are private information.

### 2.3 The depositors' game

In the decentralized economy, each depositor chooses her withdrawal strategy as part of a non-cooperative game. It will often be useful to fix the banking policy $x$ and look at the game played by depositors under that particular policy. Let $y_{-i}$ denote the profile of withdrawal strategies for all depositors except $i$. An equilibrium of this game is then defined as follows.

**Definition 1:** Given a policy $x$, an *equilibrium of the depositors' game* is a profile of strategies $\widehat{y}(x)$ such that

$$v_i\left(x, (\widehat{y}_{-i}, \widehat{y}_i)\right) \geq v_i\left(x, (\widehat{y}_{-i}, y_i)\right) \quad \text{for all } y_i, \text{ for all } i.$$

Because they are isolated, depositors do not directly observe each others' actions. Therefore, even though these actions take place sequentially, we can think of depositors as choosing their strategies simultaneously. This approach, which captures the dynamic features of sequential service in an essentially static game, was introduced by Green and Lin (2003); see Ennis and Keister (2009b) for a detailed discussion.

[7] Since only the $\pi$ impatient depositors will withdraw in period 1, the payments for $\mu > \pi$ will not occur and need not be specified.

The depositors' game has been the focus of the literature on bank runs since Diamond and Dybvig (1983). For some policies $x$, this game may not have a unique equilibrium.[8] We use $\widehat{Y}(x)$ to denote the *set* of equilibria associated with the policy $x$. We say that a *bank run* occurs in an equilibrium $\widehat{y}$ if more than $\pi$ depositors withdraw in period $1$. Since all impatient depositors will choose to withdraw in period $1$, a run occurs if and only if some patient depositors withdraw early, *i.e.*, $\widehat{y}_i(1) = 0$ for a positive measure of depositors.

## 2.4 The overall banking game

Our interest is in the interaction between depositors' withdrawal decisions and the banking policy summarized by the function $x$. We assume this policy is chosen by a benevolent banking authority, whose objective is to maximize the welfare function $U$. The banking authority is a reduced-form representation of the entire banking system of the economy, together with any regulatory agencies and other government entities that have authority over the banking system. Our analysis would be exactly the same if there were a group of profit-maximizing banks competing for deposits in period $0$ and if the authority to reschedule payments in period $1$ were held by the (benevolent) government. To keep the presentation simple, and in line with the previous literature, we present the model with this system represented by a single, consolidated entity.

We also follow the literature in permitting depositors' withdrawal decisions to be conditioned on an extrinsic "sunspot" variable that is not observed by the banking authority.[9] We assume, without any loss of generality, that the sunspot variable is uniformly distributed on $S = [0, 1]$. Each depositor then chooses a strategy $y_i : \Theta \times S \to \{0, 1\}$ in which her action is a function of the sunspot state. In equilibrium, the banking authority correctly anticipates the profile of withdrawal strategies $y$ but may not initially know the profile of actions because it does not observe the sunspot state $s$. In particular, the banking authority may not know whether a run is underway until it has observed enough actions to infer the state.

We begin our analysis with the total endowment of the economy deposited in the banking technology. One can show that if agents were allowed to choose how much of their private endowment to deposit, they would strictly prefer to deposit everything in the banking system as long as the

8 The global games approach of Carlsson and van Damme (1993) has been applied in a variety of settings to generate a unique equilibrium in this type of coordination game. As is clear from Goldstein and Pauzner (2005), however, applying this approach to the Diamond-Dybvig environment requires making strong assumptions about the investment technology and placing ad hoc restrictions on the banking policy.

9 See Diamond and Dybvig (1983), Cooper and Ross (1998), and Peck and Shell (2003).

probability of a run is low enough. In this way, our approach is without any loss of generality.

In the overall banking game, the banking authority chooses the policy $x$ to maximize welfare $U$ and each depositor chooses her withdrawal strategy $y_i$ to maximize her expected utility $v_i$. The precise definition of equilibrium in this game depends crucially on whether the banking authority commits to the policy before depositors choose their strategies.

## 2.5 Definitions of equilibrium

We now present the definition of equilibrium in the overall banking game for the environments both with and without commitment, beginning with the former.

**The environment with commitment.** We say that the banking authority has *commitment* if it chooses the entire policy $x$ before depositors make their withdrawal decisions and cannot change any part of the policy later. The previous literature has implicitly assumed commitment. Wallace (1988), for example, views the banking location as a cash machine that is programmed in advance to follow a particular payment schedule. Depositors observe the policy $x$ and, therefore, the depositors' game is a proper subgame of the "overall" banking game. The focus is, naturally, on subgame perfect equilibria, where the banking authority sets a policy $x$ with the knowledge that, in each state $s$, the withdrawal strategies will correspond to an equilibrium of the depositors' game generated by $x$. If there are multiple equilibria of the depositors' game, the banking authority must have an expectation about which equilibrium will be played; equilibrium of the overall game then requires that this expectation be correct.

We represent the banking authority's expectation of depositors' play by a selection $\widehat{y}(x,s)$ from $\widehat{Y}(x)$, that is, a function with $\widehat{y}(x,s) \in \widehat{Y}(x)$ for all $x$ and all $s$. In other words, the banking authority expects that if it chooses policy $x$, depositors will play $\widehat{y}(x,s)$ in state $s$. An equilibrium of the overall banking game obtains when the banking authority's policy choice is welfare maximizing given its expectation of depositors' play and, given this choice, the expectation is fulfilled. We formally define an equilibrium of the overall game with commitment as follows.

**Definition 2:** An *equilibrium with commitment* of the (overall) banking game is a pair $(x^*, y^*)$, together with a selection function $\widehat{y}(x,s) \in \widehat{Y}(x)$ for all $x$ and $s$, such that

$(i)\ y^*(s) = \widehat{y}(x^*, s)$ for each $s$, and

$(ii)\ \int_0^1 U(x^*, y^*(s))\, ds \geq \int_0^1 U(x, \widehat{y}(x,s))\, ds$ for all $x$.

This definition can be viewed as a type of correlated equilibrium, using a particular correlating device (which we label ‘sunspots’) that is asymmetrically observed by depositors and the banking authority. See Peck and Shell (1991) for this interpretation of correlated equilibrium.

**The environment without commitment.** In an environment without commitment, the banking authority is not able to irrevocably set the payment schedule before depositors choose their withdrawal strategies. Instead, the payment $x(\mu)$ is finally determined only when it is actually made. This approach captures important features of reality. While a banking contract is generally agreed on when funds are deposited, governments routinely reschedule payments during times of crisis. The assumption of the no-commitment case is that the rescheduling plan cannot be fixed in advance; it will be chosen as a best response to whatever situation the banking authority finds itself facing. It is worth emphasizing that the banking authority in our model is completely benevolent; its objective is always to maximize the welfare function $U$. The assumption in this case is simply that the government is unable to commit not to intervene if a crisis is underway and an improvement in resource allocation is possible.

We formalize the notion of a lack of commitment power in the following way. When choosing a payment $x(\mu)$ for some $\mu > 0$, the banking authority recognizes that, at the point this payment is made, the actions of all depositors in the order up to (and including) the current one have already been taken. Clearly these actions cannot be influenced by the choice of payment $x(\mu)$. Moreover, the banking authority cannot commit to any payments to later depositors, nor will the choice of $x(\mu)$ affect these future payments.[10] The banking authority thus considers the strategies of the remaining depositors to be independent of its choice of $x(\mu)$. In other words, in the environment without commitment, the banking authority chooses each payment $x(\mu)$ taking the entire strategy profile $y^*$ as given. This is a standard formulation of a policy game without commitment; see, for example, the discussion in Cooper (1999, p.137).

The definition of equilibrium for the environment without commitment is, therefore, as follows.

[10] With a continuum of depositors, the payment to one individual has a negligible effect on total resources and, hence, on the banking authority’s subsequent decision problems. Furthermore, the isolation of depositors implies that only the individual receiving the payment $x(\mu)$ directly observes the amount paid; all other depositors must infer the payment using the structure of equilibrium. Hence the banking authority cannot use changes in $x(\mu)$ as a “signal” aimed at influencing the behavior of depositors who have not yet learned their types and whose payments have not yet been determined.

**Definition 3:** An *equilibrium without commitment* of the (overall) banking game is a pair $(x^*, y^*)$ such that

$(i)\ y^*(s) \in \widehat{Y}(x^*)$ for all $s$, and

$(ii)\ \int_0^1 U(x^*, y^*(s))\, ds \geq \int_0^1 U(x, y^*(s))\, ds$ for all $x$.

Notice the small but important difference between Definitions 2 and 3. In the environment with commitment, the banking authority recognizes that a change in its policy will lead to a change in the behavior of depositors as specified in the function $\widehat{y}$. Without commitment, in contrast, the banking authority takes the strategies of depositors as given and must choose a best response to these strategies.

In other words, with commitment the banking authority can threaten drastic action (such as freezing all remaining deposits) when faced with a run and depositors know that this threat will be carried out if necessary. Removing the assumption of commitment imposes a form of *credibility* on the banking authority's threats; a threatened action will be deemed credible by depositors only if it is actually the banking authority's best response when faced with a run. In this way, our approach involves applying the time consistency notion of Kydland and Prescott (1977) to policies that potentially lie off of the equilibrium path of play.[11]

# 3 Equilibrium with Commitment

Diamond and Dybvig (1983) showed that the depositors' game associated with a simple demand-deposit contract has an equilibrium that achieves the first-best allocation of resources. In particular, they studied the policy

$$x(\mu) = \left\{ \begin{array}{ll} c_1^* & \text{for } \mu \in [0, \widehat{\mu}] \\ 0 & \text{otherwise} \end{array} \right\} \quad \text{with } \widehat{\mu} = (c_1^*)^{-1}, \tag{5}$$

which offers the amount $c_1^*$ to any depositor withdrawing in period 1 as long as the bank has funds. If all patient depositors wait until period 2 to withdraw, this policy will give each of them $c_2^* > c_1^*$ and, hence, each would choose to wait. The central point of their paper was that the policy in (5) generates another equilibrium of the depositors' game in which all depositors attempt to withdraw

[11] The related work of Bassetto (2005) is also concerned with the specification of government policy along potentially off-equilibrium paths and shows how multiplicity of equilibria is more common than previously thought. His approach, however, assumes commitment and only requires that announced policies be *feasible* along all possible paths of play. Condition (1) ensures feasibility in our setup; in particular, freezing deposits is always feasible. For us, the ability (or inability) to *commit* to a policy is the critical issue.

in period 1; those who arrive before the bank runs out of funds receive $c_1^*$, while those who arrive later (or who deviate and wait until period 2) receive nothing. This outcome resembles a run on the banking system and would lead to an inefficient allocation of resources.

Could a run occur in an equilibrium of the overall banking game with commitment? Diamond and Dybvig (1983) provided a partial answer to this question by showing how a deposit freeze policy could render the first-best allocation the *unique* equilibrium outcome of the depositors' game. Suppose that instead of following (5), the banking authority sets

$$x(\mu) = \left\{ \begin{array}{ll} c_1^* & \text{for } \mu \in [0, \pi] \\ 0 & \text{otherwise} \end{array} \right\}. \tag{6}$$

In other words, suppose the banking authority announces that after paying $c_1^*$ to a fraction $\pi$ of depositors in period 1, it will close its doors and refuse to serve any more depositors until period 2. Then a patient depositor will know that, regardless of how many people attempt to withdraw in period 1, the banking authority will have enough resources to pay her at least $c_2^*$ in period 2. Since $c_2^* > c_1^*$ holds, waiting to withdraw is a strictly dominant strategy for a patient depositor, and the only equilibrium of the depositors' game has $y_i(\theta_i) = \theta_i$ for all $i$, independent of the sunspot state. This policy thus costlessly eliminates the possibility of a bank run.

The above reasoning implies that any equilibrium of the overall banking game with commitment must lead to the first-best consumption allocation, with impatient depositors receiving $c_1^*$ and patient depositors receiving $c_2^*$ in all states. The banking authority's equilibrium policy $x^*$ is not uniquely defined, because many policies beside (6) will lead to the same result. In fact, the original Diamond-Dybvig policy (5) is also consistent with such an equilibrium, since it yields the same payoffs as (6) if no depositor runs. However, any profile of withdrawal strategies that has a positive measure of patient depositors withdrawing early in some states of nature is inconsistent with equilibrium, since the banking authority could raise welfare by switching to (6).

**Proposition 1** *The first-best allocation obtains in any equilibrium of the banking game with commitment.*

In other words, under the assumption of commitment, bank runs cannot occur in equilibrium because the banking authority has a policy tool – freezing all remaining deposits after $\pi$ withdrawals – that costlessly rules them out.

# 4 Equilibrium without Commitment

In earlier work (Ennis and Keister, 2009a), we showed that the deposit-freeze policy (6) is not *ex post* efficient in the event of a run. If the banking authority cannot commit to a policy, it would not choose to freeze deposits when faced with a run; doing so would deny consumption to those truly impatient depositors who have not yet been served. Depositors anticipate this fact, of course, and thus recognize that if a run were to occur, it could compromise the solvency of the banking system. In this way, the banking authority's inability to commit to a policy like (6) may generate an ex ante incentive for depositors to run.

Here we study the properties of equilibrium bank runs in the environment without commitment. We first show that the standard approach to modeling a bank run, in which *all* depositors run in some sunspot states, is inconsistent with equilibrium in our model. We then construct a class of partial-run equilibria and study the interaction between depositors' withdrawal decisions and the reactions of the banking authority in these equilibria.

It is worth pointing out that there is always an equilibrium without commitment of the overall banking game in which the first-best allocation obtains; the reasoning is almost identical to that in the commitment case. The difference between the environments with and without commitment is not related to the ability of the banking authority to generate the efficient allocation as *an* equilibrium outcome. Rather, the key difference lies in the ability – or inability – of the banking authority to rule out undesirable allocations as competing equilibrium outcomes.

## 4.1 No full-run equilibrium

In a *full* bank run, all patient depositors attempt to withdraw early in some states and wait until period 2 in the remaining states. The strategy profile associated with this type of run,

$$y_i\left(\theta_i, s\right) = \left\{ \begin{array}{ll} \theta_i & \text{for } s > s_1 \\ 0 & \text{for } s \leq s_1 \end{array} \right\} \text{ for some } s_1 \in (0,1)\,, \text{ for all } i, \tag{7}$$

has been discussed extensively in the literature; see, for example, Diamond and Dybvig (1983), Cooper and Ross (1998), and Peck and Shell (2003). Even in the environment without commitment, the model we study here cannot have a full bank run equilibrium.

**Proposition 2** *The strategy profile* (7) *cannot be part of an equilibrium of the banking game without commitment.*

To see why this is the case, consider the banking authority's best response if it were faced with the strategy profile in (7). The first $\pi$ depositors to withdraw provide no information about the state $s$, since the fraction of depositors withdrawing is at least $\pi$ in every state. The banking authority will, therefore, give some common amount $c_1$ to each of these depositors. The size of the payment $c_1$ will depend on $s_1$, of course, but the exact amount is not important for the argument.

The banking authority recognizes that after $\pi$ withdrawals have taken place, additional withdrawals in period 1 will only occur in states with $s \leq s_1$, in which case all depositors will withdraw early. The banking authority's best response will then be to set the payments $x(\mu)$ for $\mu > \pi$ so as to evenly divide its remaining assets among the remaining depositors. Each of these depositors would receive

$$x(\mu) = \frac{1 - \pi c_1}{1 - \pi} \equiv \widetilde{c}_1 \quad \text{for } \mu > \pi.$$

Given this payment schedule, does the strategy profile in (7) represent an equilibrium of the depositor's game? No; the payment available to a patient depositor who deviates and withdraws in period 2 in states $s \leq s_1$ is $R\widetilde{c}_1$, which is strictly greater than $\widetilde{c}_1$. Any patient depositor with $i > \pi$ would, therefore, prefer to wait until period 2 to withdraw. A patient depositor with $i \leq \pi$ may or may not prefer to wait, depending on the relative sizes of $c_1$ and $R\widetilde{c}_1$, but either way the strategy profile (7) is inconsistent with equilibrium behavior.

The logic above indicates that there cannot be a full run on the banking system. If, at any point in period 1, the banking authority expects all remaining depositors to withdraw early, it will react by dividing the remaining resources evenly among these depositors. This reaction removes the incentive for the depositors to run.[12] In order for a run to occur in equilibrium, therefore, depositors must follow strategies different from those in (7). In the next subsection, we show that such equilibria do indeed exist.

## 4.2 A class of partial-run equilibria

In a *partial* bank run, some patient depositors withdraw early while others do not. The following proposition shows that there exist equilibria in which partial banks runs occur in some states in the environment without commitment.

[12] Note that the banking authority is not attempting to dissuade depositors from running here; it is simply choosing a best response to depositors' actions. This feature is different from the *run-proof contracts* studied by Cooper and Ross (1998) and others, which require the banking authority to commit to a policy that removes depositors' incentive to run.

**Proposition 3** *Given $R$, $\pi$, and $\gamma$ satisfying*

$$f(\gamma,0)=\frac{R^{\frac{2-\gamma}{\gamma}}}{\pi+(1-\pi)\,R^{\frac{1-\gamma}{\gamma}}}<1 \tag{8}$$

*and given any $\lambda<1$, there exists an equilibrium of the banking game without commitment in which the fraction of depositors withdrawing in period $1$ is greater than $\lambda$ with positive probability.*

The proof, which is presented in the appendix, constructs a general class of partial-run equilibria and shows that the entire class of equilibria exist when (8) is satisfied. Notice that for any given values of $R$ and $\pi$, this condition will hold if $\gamma$ is large enough, that is, if depositors are sufficiently risk averse.

Taken together, Propositions 1 and 3 establish the importance of commitment in this model. When the banking authority can commit to follow a pre-specified banking policy, bank runs never occur in equilibrium. When this ability to commit is absent, however, run equilibria can easily exist. The bank runs in these equilibria must be partial, with only some agents participating. Moreover, the equilibria identified by Proposition 3 have a *wave structure*. If more than $\pi$ depositors withdraw in period 1, the banking authority reacts by rescheduling payments. At that point, the run may stop or it may continue. If it continues, a second wave of withdrawals will trigger another policy reaction by the banking authority. This process can repeat many times, with the number of waves determining the eventual size of the run. The next section studies this wave structure in detail.

# 5 Waves of Withdrawals and Policy Responses

A key feature of equilibrium in the environment without commitment is the interplay between depositors' withdrawal decisions and the actions of the banking authority. In the commitment case, this interplay does not arise because the banking policy is completely set before depositors choose their strategies. The choice of policy affects depositors' decisions, of course, but the effect only runs in one direction. In the environment without commitment, in contrast, the banking policy also reacts to depositors' withdrawal decisions. To understand how this interplay is captured in the wave structure of equilibrium, and to see where condition (8) comes from, it is useful to examine the simplest type of partial-run equilibrium.

## 5.1 Equilibria with a single policy response

Consider the strategy profile

$$\begin{aligned} &\text{For } s > s_1: \quad y_i(\theta_i, s) = \theta_i \qquad\qquad \text{for all } i \\ &\text{For } s \leq s_1: \quad y_i(\theta_i, s) = \left\{ \begin{array}{c} 0 \\ \theta_i \end{array} \right\} \quad \text{for} \left\{ \begin{array}{c} i \leq \pi \\ i > \pi \end{array} \right\}. \end{aligned} \tag{9}$$

In this profile, the first $\pi$ depositors in the order run on the bank in some states. Once this wave has passed, however, the run halts and only the remaining impatient depositors withdraw.[13] We construct an equilibrium based on this strategy profile in two steps. First, we derive the banking authority's best response to (9); let $\widehat{x}$ denote the best-response policy. We then show that the profile in (9) is an equilibrium of the depositors' game generated by $\widehat{x}$ whenever condition (8) holds.

**Step 1.** We calculate the banking authority's best response to (9) by working backward. First, note that the fraction of depositors withdrawing in period 1 is at most $1-(1-\pi)^2$, so the payments $x(\mu)$ for $\mu > 1-(1-\pi)^2$ need not be specified. Next, consider $x(\mu)$ for any $\mu \in \left(\pi, 1-(1-\pi)^2\right)$. These payments will only be made in states $s \leq s_{1.}$ If these payments are made, therefore, the banking authority knows that $(i)$ a run will have occurred, meaning that the first $\pi$ withdrawals were made by a mix of patient and impatient depositors, but $(ii)$ all additional withdrawals in period 1 will be made by depositors who are truly impatient. The total fraction of depositors withdrawing in period 1 will, therefore, be $1-(1-\pi)^2$.

Because depositors are risk averse, the banking authority will choose to offer a common payment to all of the (impatient) depositors who withdraw after $\pi$. We denote this payment $c_{1,2}$, where the latter subscript indicates that the payment is associated with the 2nd "stage" of the payment schedule. The banking authority will also give a common payment $c_{2,2}$ to the (patient) depositors who withdraw in period 2. These payments will be chosen to maximize the banking authority's objective function (2). Let $\psi$ denote the per-capita amount of resources the banking authority has left after the first $\pi$ withdrawals, that is,

$$\psi = \frac{1-\int_0^\pi x(\mu)\,d\mu}{1-\pi}.$$

[13] Gu (2008) studies a model with demand-deposit contracts and generates a partial-run equilibrium by having depositors observe imperfectly correlated sunspot signals. In her setting, a partial run occurs in some states and a full run in others. In our environment, in contrast, only partial runs are observed; a full run cannot occur in any state.

Then the payments $c_{1,2}$ and $c_{2,2}$ will solve

$$\max_{\{c_{1,2},c_{2,2}\}} \pi\frac{(c_{1,2})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_{2,2})^{1-\gamma}}{1-\gamma} \tag{10}$$

subject to

$$(1-\pi)c_{2,2}=R\left[\psi-\pi c_{1,2}\right].$$

Notice the similarity between this problem and (3). The strategy profile in (9) implies that when a run occurs, it halts after $\pi$ withdrawals have been made. From that point onward, only impatient depositors withdraw in period 1. The banking authority is, therefore, able to implement the first-best *continuation* allocation, given the per-capita amount $\psi$ of resources remaining. Let $(\widehat{c}_{1,2},\widehat{c}_{2,2})$ denote the solution to this problem, which will always satisfy $\widehat{c}_{2,2}>\widehat{c}_{1,2}$. Let $V(\psi)$ denote the value of the objective in (10) evaluated at the solution.

We next ask how the banking authority will set the payments to the first $\pi$ depositors who withdraw. The banking authority does not know whether these payments will go to only impatient depositors, as will happen if $s>s_1$, or to a mix of patient and impatient depositors participating in a run, as will occur if $s\le s_1$. As these withdrawals take place, the banking authority is unable to infer anything about the state $s$, since at least $\pi$ withdrawals will occur in all states. As a result, the banking authority will choose to give the same payment to all $\pi$ depositors. Any payment schedule for which $x(\mu)$ is not constant for (almost) all $\mu\le\pi$ is strictly dominated by another policy that makes the same total payment to these depositors, leaving $\psi$ unchanged, but divides the resources evenly among them.

The banking authority will, therefore, set $x(\mu)=\widehat{c}_1$ for $\mu\in[0,\pi]$, where $(\widehat{c}_1,\widehat{c}_2)$ solves

$$\max_{\{c_1,c_2\}} (1-s_1)\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma}\right)+s_1\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)V(\psi)\right) \tag{11}$$

subject to

$$\begin{aligned}(1-\pi)c_2 &= R(1-\pi c_1) \quad \text{and}\\ \psi &= \frac{1-\pi c_1}{1-\pi}.\end{aligned}$$

It is straightforward to show that $\widehat{c}_2>\widehat{c}_1$ holds. In other words, if a run does not occur (that is, if $s>s_1$), then depositors withdrawing in period 2 will receive more than depositors withdrawing in period $1$. In addition, if $s_1 > 0$, meaning that a run is possible, it can be shown that $\widehat{c}_{1,2}$ is strictly smaller than $\widehat{c}_1$ – that is, depositors who withdraw in period $1$ after it becomes clear that a partial run has taken place suffer a "discount" relative to depositors who were earlier in the order. Summarizing, the banking authority's best response to the profile of withdrawal strategies (7) is given by

$$\widehat{x}(\mu) = \left\{ \begin{array}{c} \widehat{c}_1 \\ \widehat{c}_{1,2} \end{array} \right\} \text{ for } \mu \in \left\{ \begin{array}{c} [0, \pi] \\ \left(\pi, 1 - (1-\pi)^2\right] \end{array} \right\}. \tag{12}$$

**Step 2.** We next ask if the strategy profile in (9) is an equilibrium of the depositors' game generated by $\widehat{x}$. In other words, if the banking authority were to follow the payment scheme in (12), would each depositor find it optimal to follow (9) if she believed others would do so? Impatient depositors will always choose to withdraw in period $1$, so we only need to consider the actions of patient depositors.

In states $s > s_1$, a patient depositor receives $\widehat{c}_2$ if she waits until period $2$ to withdraw, but receives $\widehat{c}_1$ if she deviates and withdraws early. Since $\widehat{c}_2 > \widehat{c}_1$ holds, waiting to withdraw is clearly the optimal choice in these states. In states $s \leq s_1$, the payment a patient depositor receives if she chooses to withdraw early depends on her index $i$. For a patient depositor with $i > \pi$, the choice is between $\widehat{c}_{1,2}$ if she withdraws early and $\widehat{c}_{2,2}$ if she waits. Since $\widehat{c}_{2,2} > \widehat{c}_{1,2}$, it is optimal for her to wait, as specified by (9).

What about patient depositors with $i \leq \pi$? Such a depositor will also receive $\widehat{c}_{2,2}$ if she waits until period $2$, but will receive the original payment $\widehat{c}_1$ if she withdraws early. She will choose to follow (9) and withdraw early if $\widehat{c}_1 > \widehat{c}_{2,2}$. The proof of Proposition 3 shows that this condition will be satisfied whenever (8) holds and the probability of a run $s_1$ is small enough. In such cases, the profile of withdrawal strategies (9) represents an equilibrium of the depositors' game generated by the policy $\widehat{x}$. Since $\widehat{x}$ is, by construction, the banking authority's best response to (9), we have constructed an equilibrium of the overall banking game without commitment. Notice that the fraction of depositors withdrawing in period $1$ in this equilibrium is stochastic: it equals $\pi$ in some states and $1 - (1-\pi)^2$ in others.

This construction sheds some light on the form of condition (8). When a run occurs, the banking authority will pay out a total of $\pi\widehat{c}_1$ to withdrawing depositors before inferring that a run has taken place. When $\pi$ is large, therefore, the banking authority makes this inference relatively late in the course of events, after a large number of withdrawals have taken place and the remaining resources are relatively small. This fact increases the ex ante incentive for depositors to run and, as a result, condition (8) is more likely to hold. A similar effect arises when depositors are very risk averse (i.e., $\gamma$ is large), because the efficient allocation offers depositors a high level of liquidity insurance and, hence, $\widehat{c}_1$ is relatively large. As mentioned above, for any given values of $R$ and $\pi$, condition (8) is always satisfied when depositors are sufficiently risk averse.

### 5.2 Multiple waves of withdrawals and responses

Proposition 3 states that, when condition (8) holds, there exist equilibria in which the fraction of depositors withdrawing in period 1 is very close to one with positive probability. The proof in the appendix constructs such equilibria in closed form. The basic idea underlying the proof, however, can be understood by extending the example presented in the previous subsection. In this subsection, we describe an equilibrium in which the run may continue even after the banking authority has rescheduled payments. Once this is done, it will be fairly easy to see how the logic can be extended further to deliver the proof of Proposition 3.

Consider the following profile of withdrawal strategies:

$$\begin{array}{lll}
\text{for } s \geq s_1 : & y_i(\theta_i, s) = \theta_i & \text{for all } i \\[1em]
\text{for } s \in [s_2, s_1) : & y_i(\theta_i, s) = \left\{ \begin{array}{c} 0 \\ \theta_i \end{array} \right\} & \text{for} \left\{ \begin{array}{c} i \leq \pi \\ i > \pi \end{array} \right\} \\[1em]
\text{for } s < s_2 & y_i(\theta_i, s) = \left\{ \begin{array}{c} 0 \\ \theta_i \end{array} \right\} & \text{for} \left\{ \begin{array}{c} i \leq 1-(1-\pi)^2 \\ i > 1-(1-\pi)^2 \end{array} \right\}
\end{array} \tag{13}$$

for some $s_1 > s_2 > 0$. Notice how early withdrawals have a *wave structure* in this strategy profile. When $s < s_1$, there is a wave of early withdrawals as the first $\pi$ depositors run on the bank, exactly as in the previous subsection. At this point, the run may halt or it may continue. In particular, if $s < s_2$ another wave of early withdrawals will take place as the next group of depositors run. Following this wave, the run necessarily halts and the only further withdrawals are those made by the remaining impatient depositors.

To construct an equilibrium based on this strategy profile, we follow the same two steps as in the previous case. First, we derive the banking authority's best response to (13), denoted $\widehat{x}$, and then we show that (13) is an equilibrium of the depositors' game based on $\widehat{x}$ whenever (8) holds.

**Step 1.** Without going into the details of the calculations, it is easy to see that the banking authority's best response to the strategy profile in (13) must be of the form

$$\widehat{x}(\mu) = \left\{ \begin{array}{c} c_1 \\ c_{1,2} \\ c_{1,3} \end{array} \right\} \text{ for } \left\{ \begin{array}{l} \mu < \pi \\ \mu \in \left(\pi, 1-(1-\pi)^2\right] \\ \mu > 1-(1-\pi)^2 \end{array} \right\}. \tag{14}$$

The reasoning behind (14) is exactly the same as that behind the policy in (12). As the first $\pi$ withdrawals are taking place, the banking authority is unsure whether or not a run is underway and it will choose to offer a common payment $c_1$ on all of these withdrawals. This payment can be found by solving a problem similar to (11); see the proof of Proposition 3 in the appendix for details.

If more than $\pi$ withdrawals take place in period 1, the banking authority will recognize that a run is underway and will reschedule payments. At this point, however, the banking authority is unsure whether the run will halt, with all additional period-1 withdrawals being made by impatient depositors, or if it will continue. The run will halt if $s \in [s_2, s_1)$ and will continue if $s < s_2$; hence, the banking authority assigns conditional probability $s_2/s_1$ to the event that the run continues. Based on this probability, the banking authority will choose to give a common payment $c_{1,2}$ to the next $\pi(1-\pi)$ depositors who withdraw. If more than $1-(1-\pi)^2$ withdrawals take place in period $1$, the banking authority will be able to infer that $s < s_2$. In this case it will solve a problem similar to (10) to find the best payment $c_{1,3}$.

**Step 2.** The remaining question is whether or not the withdrawal strategies (13) are an equilibrium of the depositors' game generated by the policy (14). Would each individual depositor be willing to follow the strategy in (13) if she expected all others to do so? The answer will be affirmative if and only if the payments induced by the policy (14) satisfy

$$c_1 \leq c_2, \quad c_{1,2} \leq c_{2,2}, \text{ and } c_{1,3} \leq c_{2,3}, \tag{15}$$

as well as

$$c_1 \geq c_{2,2}, \quad c_1 \geq c_{2,3}, \text{ and } c_{1,2} \geq c_{2,3}. \tag{16}$$

The inequalities in (15) guarantee that if a run is not currently underway when a patient depositor has the opportunity to withdraw, she will be willing to wait until period 2. The first inequality applies to states $s \geq s_1$, where each depositor receives $c_1$ if she withdraws in period $1$ and $c_2$ if she waits until period $2$. The second applies to states $s \in [s_2, s_1)$ and depositors $i > \pi$, while the third inequality applies to states $s < s_2$ and depositors $i > 1 - (1 - \pi)^2$ . It can be shown that these inequalities always hold.

The inequalities in (16) guarantee that a patient depositor is willing to participate in the run if one is underway when she has the opportunity to withdraw. The first inequality guarantees that depositors with $i \leq \pi$ are willing to run in states $s \in [s_2, s_1)$ , while the second ensures that these same depositors are willing to run in states $s < s_2$. The third inequality guarantees that depositors with $i$ between $\pi$ and $1 - (1 - \pi)^2$ are willing to run in states $s < s_2$. Whether or not the inequalities in (16) hold will depend on the cutoff states $s_1$ and $s_2$, which have a large impact on the payments that the banking authority chooses. It can be shown that there exist $s_1 > s_2 > 0$ such that all of these inequalities hold if and only if condition (8) holds. This reasoning shows that, under condition (8), there exist equilibria in which the fraction of depositors withdrawing in period 1 is equal to $1 - (1 - \pi)^3$ with positive probability.

Nothing in the logic presented above requires a run to end with certainty after a second wave of early withdrawals. This same approach can be used to construct equilibria in which a run may occur in any finite number of waves, each of which elicits a policy response from the banking authority. In this way, an equilibrium can be constructed in which the fraction of depositors withdrawing in period $1$ is very close to one in some states. The details of this construction can be found in the proof given for Proposition 3 in the appendix.

### 5.3 Discussion

An interesting feature of the equilibria identified in Proposition 3 is that, even if nearly all depositors end up withdrawing in period 1, the banking authority remains "optimistic" throughout the period that the run has already ended. As discussed above, if the banking authority ever believed that a full run was underway, it would reschedule payments in such a way that the remaining depositors would choose not to run. The only way a run can continue (or even start) is if the banking authority is fairly optimistic and, therefore, sets the payment for early withdrawals relatively high. This fact implies that bank runs *must* occur in waves in our environment, with the run likely to end after each wave.

This feature of the model is reminiscent of events in the summer of 2008, after the collapse of Bear Stearns but before the failure of Lehman Brothers. Policy makers around the world had implemented a variety of responses to the financial crisis. More drastic actions could have been taken and may – with the benefit of hindsight – have prevented the crisis from deepening. The fact that these actions were not taken appears to have reflected, in part, a belief that the worst of the crisis may have already passed.[14] Our model indicates that this feature is an essential element of financial crises. The policy maker correctly anticipates the probability that conditions will worsen and responds appropriately. When this probability is small enough, however, the response leaves the door open for the crisis to deepen. In this way, the model illustrates how optimism about the course of events can combine with limited commitment to lay the seed of a deepening financial crisis.

Peck and Shell (2003) study a model with aggregate uncertainty about the fraction of impatient depositors and construct examples of equilibria in which all depositors run. In these equilibria, the banking authority remains optimistic that it is observing an unusually large realization of the fraction of impatient depositors rather than a run and, hence, believes that the withdrawals will likely stop soon. In this sense, the aggregate uncertainty in their model plays the role of the wave structure of equilibrium in ours.

The two approaches have fundamental differences, however. In their setting, the banking authority can never know for certain whether or not a run has occurred, even after the fact. In the examples they construct, the event in which *all* depositors are impatient is much more likely than a run. We do not believe it is plausible to characterize events in the U.S. in the early 1930s or in Argentina in 2001 as possibly resulting from a spike in the fundamental demand for liquidity. Once underway, a run on the banking system is easily recognized. Our model has this property: when more than $\pi$ withdrawals take place, the banking authority correctly infers that a run has taken place. Its optimism is not about whether a run has *occurred*, but rather about whether the run will *continue* after payments are rescheduled.

[14] See, for example, a speech given by then-Governor Mishkin on July 2, 2008: "The period of extreme stress seems to have abated, and financial markets are showing some tentative signs of revival." Available at: http://www.federalreserve.gov/newsevents/speech/mishkin20080702a.htm

# 6 Concluding Remarks

The issues of commitment, credibility, and time-inconsistency are pervasive in economics and have been studied extensively. In banking theory, however, the importance of these issues has received relatively little attention, apart from often informal treatments of bank bailouts. [15] Recent events have highlighted the difficulty policy makers would face in trying to follow a pre-specified course of action throughout a financial crisis. Government officials and central bankers have repeatedly described and justified their actions during the financial crisis as the best available response to the situation they faced, rather than as the result of a pre-formulated plan. In such situations, the anticipated policy response to a crisis clearly influences people's *ex ante* incentives and behavior.

The paper analyzes the role of commitment in banking policies designed to respond to the possibility of a run on the banking sector. We study a setting in which bank runs would never occur under commitment because, in that case, the threat to freeze deposits in the event of a run convinces depositors not to run in the first place. In contrast, equilibrium bank runs can easily occur in this same setting when policy makers are unable to commit to future actions.

Moreover, equilibrium bank runs in our model take an interesting, and perhaps realistic, form. A run is necessarily partial, with only some depositors participating. The policy maker in our model observes waves of withdrawals from the banking system during a crisis. After each wave, the policy maker reacts based, in part, on how likely she thinks it is that the run will continue. This reaction, and depositors' anticipation of the reaction, affects the incentive for depositors to participate in the run. In this way, the model illustrates how the interplay between the actions of depositors and the responses of the policy maker shapes the course of a financial crisis.

This structure also implies that the size of the crisis in our model is stochastic. After each wave of withdrawals, the crisis may end or it may deepen as an additional wave of withdrawals takes place, leading to an even stronger response from the policy maker. An immediate implication of this structure is that larger crises are less likely to occur than smaller ones. In addition, the structure of the model requires that the policy maker always be optimistic that the worst of the crisis has likely passed. This optimism prevents the policy maker from choosing a more drastic response; the less-drastic response, in turn, is what leaves open the possibility that the crisis will continue.

[15] Two notable exceptions are Mailath and Mester (1994) and Acharya and Yorulmazer (2007), both of which deal with credibility issues in policies regarding bank closure.

A large number of papers have addressed applied questions related to bank runs and financial crises using versions of the Diamond-Dybvig model.[16] In order to obtain a run equilibrium in a tractable way, these papers place ad hoc restrictions on the banking contract, such as requiring banks to redeem deposits at face value until their assets are totally depleted. This approach has obvious drawbacks, including the fact that the results of such an exercise may depend critically on what restrictions are imposed. The model presented here offers an alternative. There are no restrictions on contracts other than those imposed by the physical environment, and yet the model is highly tractable. In addition, the model captures the interplay between withdrawal decisions and policy responses in a way that was absent in the previous literature.

The main lesson of the paper is that the inability of policy makers to commit to a future course of action in the event of a crisis may lie at the root of the problem of financial fragility. This insight suggests that strong institutions, which limit the flexibility of policy makers during a financial panic, may have a stabilizing influence on the financial system. It also suggests that developing and empowering such institutions may produce important benefits that have not been previously recognized in the economics literature.

[16] See, for example, Temzelides (1997), Cooper and Ross (1998), Allen and Gale (2000), Chang and Velasco (2001), Ennis and Keister (2003), Goldstein and Pauzner (2005), and Uhlig (2010) to name only a few.

# Appendix A. Proof of Proposition 3

**Proposition 3:** *If* (8) *holds, then for any* $\lambda < 1$ *there exists an equilibrium of the banking game without commitment in which the fraction of depositors withdrawing in period* $1$ *is greater than* $\lambda$ *with positive probability.*

The proof is constructive. Let $K$ be the smallest integer such that

$$1 - (1-\pi)^{K+1} > \lambda$$

holds. Consider the strategy profile

$$\begin{array}{lll} \text{for } s \geq s_1 : & y_i(\theta_i, s) = \theta_i & \text{for all } i \\ \text{for } s \in [s_{k+1}, s_k) : & y_i(\theta_i, s) = \left\{ \begin{array}{c} 0 \\ \theta_i \end{array} \right\} & \text{for } i \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} 1 - (1-\pi)^k \end{array} \tag{17}$$

for $k = 1, \ldots K$, where

$$1 > s_1 > \ldots > s_K > s_{K+1} \equiv 0.$$

Under this strategy profile, the fraction of depositors withdrawing in period $1$ is $1-(1-\pi)^{K+1}$ with probability $s_K > 0$. Therefore, if we can show that (17) is part of an equilibrium of the banking game without commitment, the proposition will be proved. We break this task into two steps, which are addressed in separate lemmas below. First, Lemma 1 derives the banking authority's best response to this strategy profile, which we denote $\widehat{x}$. Lemma 2 then shows that when (8) holds, we can choose the numbers $s_k$ such that (17) is an equilibrium of the depositors' game generated by $\widehat{x}$. The result in the proposition follows immediately from these two lemmas.

**Lemma 1** *The banking authority's best response to* (17) *is*

$$\widehat{x}(\mu) = \left( \prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j} \right) \frac{1}{A_k} \ \ \textit{for } \mu \in \left( 1 - (1-\pi)^{k-1}, 1 - (1-\pi)^k \right],$$

*where*

$$A_k = \left( (1-q_k) R^{1-\gamma} + q_k \left( \pi + (1-\pi) A_{k+1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}, \quad \textit{for } k = 1, \ldots, K+1. \tag{18}$$

**Proof:** We work backwards. Define $\psi_K$ to be the per-capita resources remaining after $1-(1-\pi)^K$

withdrawals have been made, that is,

$$\psi_K = \frac{1 - \int_0^{1-(1-\pi)^K} x(\mu)\, d\mu}{(1-\pi)^K}.$$

We first derive the payments $x(\mu)$ for $\mu \in \left(1-(1-\pi)^K, 1-(1-\pi)^{K+1}\right]$.[17] The banking authority recognizes that under (17) these payments will only be made in states $s < s_K$ and that all of these payments, in the event they are made, will go to impatient depositors. The remaining patient depositors will wait until period 2 to withdraw. Because depositors are risk averse, the banking authority will choose to give the same amount to all impatient depositors; we denote this amount $c_{1,K+1}$, where the latter part of the subscript indicates that these payments would apply after there have been $K$ waves of withdrawals and the run has halted. Let $c_{2,K+1}$ denote the payment that the remaining patient depositors will receive in period 2. These payment amounts will be chosen to solve

$$\max_{c_{1,K+1}, c_{2,K+1}} \pi \frac{(c_{1,K+1})^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(c_{2,K+1})^{1-\gamma}}{1-\gamma} \tag{19}$$

subject to

$$(1-\pi) c_{2,K+1} = R\left[\psi_K - \pi c_{1,K+1}\right]$$

and non-negativity constraints. Notice that this problem resembles that for finding the first-best allocation, but with per-capita resources set to $\psi_K$ instead of 1. The solution is

$$\widehat{c}_{1,K+1} = \psi_K \frac{1}{\pi + (1-\pi) A_{K+1}} \quad \text{and} \quad \widehat{c}_{2,K+1} = \psi_K \frac{R A_{K+1}}{\pi + (1-\pi) A_{K+1}}, \tag{20}$$

where

$$A_{K+1} \equiv R^{\frac{1-\gamma}{\gamma}} < 1. \tag{21}$$

Let $V_{K+1}$ denote the value of the objective in (19) evaluated at the solution, that is

$$V_{K+1}(\psi_K) = \pi \frac{(\widehat{c}_{1,K+1})^{1-\gamma}}{1-\gamma} + (1-\pi) \frac{(\widehat{c}_{2,K+1})^{1-\gamma}}{1-\gamma},$$

or, substituting in (20),

$$V_{K+1}(\psi_K) = (\pi + (1-\pi) A_{K+1})^{\gamma} \frac{(\psi_K)^{1-\gamma}}{1-\gamma}.$$

[17] Under (17), there are no circumstances in which the payments associated with $\mu \geq 1-(1-\pi)^{K+1}$ will be made. The best-response levels for these payments are, therefore, indeterminate and do not matter for our analysis.

Next, we consider the payments in the interval

$$\mu \in \left(1-(1-\pi)^{k-1}, 1-(1-\pi)^{k}\right] \quad \text{for any } k \in \{1,\ldots,K\}.$$

These payments will be made in states $s \leq s_{k-1}$. Unlike in the previous case, the banking authority is not sure if these payments will go only to impatient depositors, as will occur if $s \in [s_k, s_{k-1})$, or to a mix of patient and impatient depositors during a continued run, as will occur if $s < s_k$. Regardless of which case applies, however, the banking authority will want to give the same payment to all depositors who withdraw in this interval. In other words, any payment schedule for which $x(\mu)$ is not constant for (almost) all $\mu$ in this interval is strictly dominated by another policy that makes the same total payments to these depositors, but divides the resources evenly among them. Let $c_{1,k}$ denote the payment given to depositors withdrawing in this interval in period 1. Let $c_{2,k}$ denote the payment that will be received by patient depositors in period 2 if there are no further withdrawals in period 1, that is, if $s \in [s_k, s_{k-1})$.

Before we write the optimization problem for choosing these payment levels, we introduce some notation to simplify the statement of the problem. First, define $\psi_{k-1}$ to be the amount of resources per capita that remain after $1-(1-\pi)^{k-1}$ withdrawals in period 1, that is,

$$\psi_{k-1} = \frac{1-\int_0^{1-(1-\pi)^{k-1}} x(\mu)\, d\mu}{(1-\pi)^{k-1}} \quad \text{for } k = 1,\ldots,K.$$

Straightforward calculations then yield the following relationship between $\psi_{k-1,}$ the payments $c_{1,k}$, and the per-capita resources $\psi_k$ remaining after these payments are made,

$$\psi_k = \frac{\psi_{k-1} - \pi c_{1,k}}{1-\pi}. \tag{22}$$

Next, define

$$q_k = \frac{s_k}{s_{k-1}} = \text{Prob}\left[s < s_k \mid s < s_{k-1}\right] \quad \text{for } k = 1,\ldots,K,$$

with $s_0 \equiv 1$. In other words, $q_k$ is the probability that the run will continue into the $k^{\text{th}}$ wave, given that it has lasted for $k-1$ waves. Finally, let $V_k\left(\psi_{k-1}\right)$ denote the average expected utility of depositors with $i > 1-(1-\pi)^{k-1}$ conditional on $s < s_{k-1}$. In other words, $V_k$ measures the expected utility of depositors who have not yet been served when the banking authority discovers that the run has at least $k-1$ waves. Then the banking authority will choose the payment $c_{1,k}$ to solve

$$\max_{c_{1,k},c_{2,k}} (1-q_k)\left(\pi\frac{(c_{1,k})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_{2,k})^{1-\gamma}}{1-\gamma}\right)+q_k\left(\pi\frac{(c_{1,k})^{1-\gamma}}{1-\gamma}+(1-\pi)V_{k+1}(\psi_k)\right)$$

subject to

$$(1-\pi)c_{2,k}=R\left[\psi_{k-1}-\pi c_{1,k}\right],$$

(22), and non-negativity constraints. The first term in the objective function represents utility in the event that the run halts after $k-1$ waves. In this case, the remaining impatient depositors all receive $c_{1,k}$ and the remaining patient depositors receive $c_{2,k}$ in period $2$. The second term represents utility in the event that the run continues into the $k^{\text{th}}$ wave, which occurs with probability $q_k$. In this case, the first $\pi$ depositors to withdraw (a mix of impatient and patient depositors) will receive $c_{1,k}$. The remaining depositors will receive payments after the next phase of the policy response takes effect; the utility of these depositors is captured by the value function $V_{k+1}$.

Solving this problem recursively backward, substituting the value function for each value of $k$ into the problem for $k-1$ yields

$$\widehat{c}_{1,k}=\psi_{k-1}\tfrac{1}{\pi+(1-\pi)A_k},\quad \widehat{c}_{2,k}=\psi_{k-1}\tfrac{RA_k}{\pi+(1-\pi)A_k},\quad \text{and}$$

$$V_k\left(\psi_{k-1}\right)=\left(\pi+(1-\pi)A_k\right)^{\gamma}\tfrac{\left(\psi_{k-1}\right)^{1-\gamma}}{1-\gamma},$$

where $A_k$ is given in (18). We can then replace the $\psi_k$ terms as follows. Since $\psi_0=1$ (by definition), we have

$$\widehat{c}_{1,1}=\frac{1}{\pi+(1-\pi)A_1}.$$

Then we can calculate the amount of resources remaining after the first $\pi$ withdrawals

$$\psi_1=\frac{1-\pi\widehat{c}_{1,1}}{1-\pi}=\frac{A_1}{\pi+(1-\pi)A_1},$$

and use this amount to find the optimal payment levels following the first policy response

$$\widehat{c}_{1,2}=\frac{A_1}{\pi+(1-\pi)A_1}\frac{1}{\pi+(1-\pi)A_2}\quad\text{and}\quad \widehat{c}_{2,2}=\frac{A_1}{\pi+(1-\pi)A_1}\frac{A_2}{\pi+(1-\pi)A_2}R.$$

Continuing this process forward yields

$$\psi_k=\prod_{j=1}^{k}\frac{A_j}{\pi+(1-\pi)A_j}$$

and

$$\widehat{c}_{1,k} = \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j}\right) \frac{1}{A_k} \text{ and } \widehat{c}_{2,k} = \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j}\right) R, \tag{23}$$

which establishes the Lemma. ■

**Lemma 2** *If* (8) *holds, there exist* $1 > s_1 > \ldots s_K > 0$ *such that* (17) *is an equilibrium of the depositors' game generated by* $\widehat{x}$.

**Proof:** Since impatient depositors will always choose to withdraw early, we only need to check the optimal behavior of a depositor when she is patient. The strategies in (17) are individually optimal if

$$\left.\begin{array}{lll} (a) & \widehat{c}_{1,j} \geq \widehat{c}_{2,k} & \text{for } j = 1, \ldots k-1 \\ (b) & \widehat{c}_{1,k} \leq \widehat{c}_{2,k} & \end{array}\right\} \quad \text{for } k = 1, \ldots, K+1.$$

The inequalities on line $(a)$ imply that patient depositors are willing to participate in the run. If the run lasts for $k-1$ waves, then a patient depositor who chooses not to run will receive $\widehat{c}_{2,k}$. A patient depositor who withdraws early receives $\widehat{c}_{1,j}$ for some $j < k$ that depends on her index $i$. If each of these inequalities hold, then all patient depositors who have an opportunity to withdraw during the run will choose to do so. The inequalities on line $(b)$ are often referred to as the *incentive compatibility* constraint. They imply that if a run is not underway, or has halted before a depositor is served, then a patient depositor will be willing to wait and withdraw in period 2.

We examine line $(b)$ first. From (21) we have $RA_{K+1} = R^{\frac{1}{\gamma}} > 1$. Then, using (18), we have

$$RA_k = \left((1-q_k) R + q_k \left(\pi R + (1-\pi) RA_{k+1}\right)^{\gamma}\right)^{\frac{1}{\gamma}}, \quad \text{for } k = 1, \ldots, K.$$

Applied recursively from $k = K$ down to $k = 1$, this expression demonstrates that

$$RA_k > 1 \quad \text{for } k = 1, \ldots, K+1.$$

It then follows immediately from (23) that $(b)$ holds.

Next, we examine line $(a)$. First, from (23) we have

$$\widehat{c}_{1,j+1} = \frac{A_j}{\pi + (1-\pi) A_{j+1}} \widehat{c}_{1,j}. \tag{24}$$

It is straightforward to show that

$$\frac{A_j}{\pi + (1-\pi)\, A_{j+1}} < 1 \quad \text{for } j = 1, \ldots, K.$$

Equation (24) therefore shows that the payment received by depositors in each wave is smaller than in the previous wave, an intuitive result. More importantly, this result also implies that instead of checking the $k-1$ inequalities on line $(a)$ for each value of $k$, we only need to check the last one:

$$\widehat{c}_{1,k-1} \geq \widehat{c}_{2,k} \quad \text{for } k = 2, \ldots, K+1.$$

This inequality can be written as

$$\widehat{c}_{1,k-1} = \left( \prod_{j=1}^{k-1} \frac{A_j}{\pi + (1-\pi)\, A_j} \right) \frac{1}{A_{k-1}} \geq \left( \prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi)\, A_j} \right) R = \widehat{c}_{2,k},$$

which can be reduced to

$$(A_k R)^{\gamma} \left( (1 - q_{k-1}) \frac{R^{1-\gamma}}{(\pi + (1-\pi)\, A_k)^{\gamma}} + q_{k-1} \right) < 1 \quad \text{for } k = 2, \ldots, K+1. \tag{25}$$

By replacing the $A_k$ terms recursively, using (18), we have $K$ inequalities involving only the parameters $R$, $\gamma$, $\pi$, and the (endogenous) probabilities $q_1, \ldots, q_K$. The question is under what conditions these probabilities can be chosen so that all $K$ inequalities hold.

Suppose we set $q_k = 0$ for all $k$. Then $A_k = R^{\frac{1-\gamma}{\gamma}}$ for all $k$ and (25) reduces to the same inequality for all values of $k$ :

$$R^{\frac{1}{\gamma}} \frac{R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi)\, R^{\frac{1-\gamma}{\gamma}}} < 1,$$

which is exactly condition (8). Since the inequalities (25) are clearly continuous in the variables $q_k$, we therefore know that when (8) holds, there exists a number $q > 0$ such that (25) holds for all $k$ if we set $q_k = q$ for all $k$. We can then back out the cutoff states $s_1, \ldots, s_K$ by

$$\begin{aligned} s_1 &= q \\ s_k &= q s_{k-1} = q^k \quad \text{for } k = 2, \ldots, K. \end{aligned}$$

Since $s_K > 0$ holds, we have established the lemma. ■

## References


Acharya, Viral V. and Tanju Yorulmazer (2007) "Too many to fail: an analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation* 16: 1-31.

Al-Najjar, Nabil I. (2004) "Aggregation and the law of large numbers in large economies," *Games and Economic Behavior* 47: 1-35.

Albanesi, Stefania, V.V. Chari, and Lawrence J. Christiano (2003) "Expectation traps and monetary policy," *Review of Economic Studies* 70: 715-741.

Allen, Franklin and Douglas Gale (2000) "Financial contagion," *Journal of Political Economy* 108: 1-33.

Bassetto, Marco (2005) "Equilibrium and government commitment," *Journal of Economic Theory* 124: 79-105.

Carlsson, Hans and Eric van Damme (1993) "Global games and equilibrium selection," *Econometrica* 61: 989-1018.

Chang, Roberto and Andres Velasco, "A model of financial crises in emerging markets," *Quarterly Journal of Economics* 116 (2001), 489-517.

Chari, V.V. and R. Jagannathan (1988) "Banking Panics, Information, and Rational Expectations Equilibrium," *Journal of Finance* 43: 749-761.

Cooper, Russell, *Coordination Games: Complementarities and Macroeconomics*, Cambridge University Press, 1999.

Cooper, Russell and Thomas W. Ross (1998) "Bank runs: liquidity costs and investment distortions," *Journal of Monetary Economics* 41: 27-38.

de Nicolò, Gianni (1996) "Run-proof banking without suspension or deposit insurance," *Journal of Monetary Economics* 38: 377-390.

Diamond, Douglas W. and Phillip H. Dybvig (1983) "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy* 91: 401-419.

Engineer, Merwan, (1989) "Bank runs and the suspension of deposit convertibility," *Journal of Monetary Economics* 24: 443-454.

Ennis, Huberto M. and Todd Keister (2003) "Economic growth, liquidity, and bank runs," *Journal of Economic Theory* 109: 220-245.

Ennis, Huberto M. and Todd Keister (2009a) "Bank runs and institutions: The perils of intervention," *American Economic Review* 99: 1588-1607.

Ennis, Huberto M. and Todd Keister (2009b) "Run equilibria in the Green-Lin model of financial intermediation," *Journal of Economic Theory* 144: 1996-2020.

Glomm, Gerhard and B. Ravikumar (1995) "Endogenous public policy and multiple equilibria," *European Journal of Political Economy* 11: 653-662.

Goldstein, Itay and Ady Pauzner (2005) "Demand deposit contracts and the probability of bank runs," *Journal of Finance* 60: 1293-1327.

Gorton, Gary (1985) "Bank suspension of convertibility," *Journal of Monetary Economics* 15: 177-193.

Green, Edward J. and Ping Lin (2000) "Diamond and Dybvig's classic theory of financial intermediation: What's missing?" Federal Reserve Bank of Minneapolis *Quarterly Review* 24: 3-13.

Green, Edward J. and Ping Lin (2003) "Implementing efficient allocations in a model of financial intermediation," *Journal of Economic Theory* 109: 1-23.

Gu, Chao (2008) "Asymmetric information and bank runs," mimeo., University of Missouri.

King, Robert G. (2006) "Discretionary policy and multiple equilibria," Federal Reserve Bank of Richmond *Economic Quarterly* 92: 1-15.

King, Robert G. and Alexander L. Wolman (2004) "Monetary discretion, pricing complementarity, and dynamic multiple equilibria," *Quarterly Journal of Economics* 119: 1513-1553.

Kydland, Finn E. and Edward C. Prescott (1977) "Rules rather than discretion: the inconsistency of optimal plans," *Journal of Political Economy* 85: 473-492.

Mailath, George J. and Loretta J. Mester (1994) "A positive analysis of bank closure," *Journal of Financial Intermediation* 3: 272-299.

Peck, James and Karl Shell (1991) "Market uncertainty: correlated and sunspot equilibria in imperfectly competitive economies," *Review of Economic Studies* 58: 1011-1029.

Peck, James and Karl Shell (2003) "Equilibrium bank runs," *Journal of Political Economy* 111: 103-123.

Temzelides, Theodosios (1997) "Evolution, coordination, and banking panics," *Journal of Monetary Economics* 40: 163-183.

Uhlig, Harald (2010) "A model of a systemic bank run," *Journal of Monetary Economics*, in press.

Wallace, Neil (1988) "Another attempt to explain an illiquid banking system: the Diamond and Dybvig model with sequential service taken seriously," *Federal Reserve Bank of Minneapolis Quarterly Review* 12 (Fall): 3-16.

Wallace, Neil (1990) "A banking model in which partial suspension is best," *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Fall): 11-23.