Allocating Losses: Bail-ins, Bailouts and Bank Regulation*

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Abstract

We study the interaction between the government's bailout policy and a bank's willingness to impose losses on (or "bail in") investors based on its private information. In the absence of regulation, bail-ins in the early stages of a crisis are too small, while bailouts are too large and too frequent. Moreover, the bank may face a run by informed investors, creating further distortions and leading to a larger bailout. We show how a regulator with limited information can raise welfare and, in some cases, improve financial stability. The optimal policy involves partial delegation: the regulator imposes some restrictions, but allows the bank to choose its bail-in within these bounds.

Keywords: Bank bailouts, moral hazard, financial stability, banking regulation

JEL Codes: E61, G18, G28

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1 Introduction

In periods of crisis, banks and other financial institutions suffer losses that are eventually borne in some combination by their own investors and creditors and, possibly, by the public sector in the form of a bailout. How these losses are allocated between private agents and the public sector has important implications for incentives and behavior in normal times as well as for the allocation of resources in society. After the global financial crisis of 2008 and the subsequent European debt crisis, a broad consensus emerged that too many of the losses in these events fell on the public sector, that is, bailouts were too frequent and too large. This perception led policy makers to draft rules requiring financial institutions to impose more losses on (or "bail in") their investors/creditors in future crises. How effective these mechanisms will be in practice remains to be seen. Even at a conceptual level, however, it is not well understood how losses *should* be allocated in a crisis, nor what types of bail-in policies are likely to be most effective.

We study the interaction between bail-ins and bailouts, focusing on the early stages of a crisis. Our model builds on the classic framework of Diamond and Dybvig (1983), where investors face idiosyncratic liquidity risk and pool their resources in a bank.[1] Bank assets are risky in our model and the size of the bank's loss during a crisis is initially not known to policy makers. Some of the bank's creditors have private information about this loss and can withdraw before the information is revealed. The bank has the ability to bail in these creditors by paying them less than in normal times. In practice, this bail-in represents any action that preserve resources within the bank, including lowering dividend payments, restricting withdrawals and/or imposing withdrawal fees. We study banks' incentives in making this bail-in decision and ask if regulating its choice can improve welfare.

Our model provides a framework for evaluating policies like the reforms to money market mutual funds adopted in the U.S. in 2014.[2] Under these rules, some funds are permitted to limit redemptions and impose withdrawal fees – a type of bail-in – during periods of financial stress. A fund is directed to take these actions if doing so is in the best interests of its investors. This policy raises interesting questions: What are the best interests of an institution's investors in such a situation? Are these rules likely to achieve desirable

[1] While we use the word *bank* throughout the paper, our analysis also applies to many intermediation arrangements outside of commercial banks that perform maturity or liquidity transformation. Yorulmazer (2014) discusses several such arrangements that encountered disruptions during the financial crisis of 2008. See also Chen et al. (2010) for an analysis of open-end mutual funds, Schmidt et al. (2016) on money market mutual funds, and Goldstein et al. (2017) on corporate bond funds.

[2] See Ennis (2012) for a discussion of the fundamental issues involved in reforming money market mutual funds. Ennis et al. (2023) provide a critical overview of the 2014 reforms in the U.S. and discuss recent proposals for future reform.

outcomes? Another example is the debate over whether regulators should restrict dividend payments by banks during events like the onset of the Covid-19 pandemic. The European Central Bank recommended on March 27, 2020, that banks "refrain from making dividend distributions and performing share buy-backs aimed at remunerating shareholders" during this period.[3] In the U.S., the Federal Reserve moved on June 25, 2020, to prohibit share repurchases and to cap dividend payments by large banks. When is it desirable to impose broad restrictions on the payments banks make to their investors? What types of restrictions are most effective? We develop a model to address these questions.

The efficient allocation of a bank's loss in our model depends on the public sector's cost of funds. If this cost is high enough, a benevolent planner will provide no bailouts and will cover the entire loss by bailing in the bank's investors. When this cost is lower, however, the planner will provide a bailout if the bank's loss is sufficiently large. In other words, the planner wants the public sector to absorb some of the "tail risk" in the economy, which implies that a combination of bail-ins and bailouts is efficient. In both cases, the planner will impose the same bail-in on all investors, regardless of whether they withdraw early or remain invested in the bank.

In the decentralized economy, the bank's incentive to bail in early-withdrawing investors depends on what bailout policy it expects. We assume the government cannot commit; it will choose the bailout as a best response to the situation at hand when the bank's loss is revealed. This situation, in turn, will depend on the bail-in that was applied to the investors who have already withdrawn at that point. In particular, a smaller initial bail-in leaves the bank in worse condition, leading to a larger bailout. This pattern clearly distorts the bank's incentive when choosing the initial bail-in. We show that, in states where the bank anticipates being bailed out, its initial bail-in is always smaller than the planner would choose and the subsequent bailout is always larger.

The smaller initial bail-in can also lead to a run on the bank in some states, which would never occur in the planner's allocation. When the bank chooses a smaller bail-in, it increases the incentive for investors to withdraw early, before policy makers observe the bank's loss and the bail-in increases to the efficient level. We show that, in some cases, withdrawing early becomes a dominant strategy and leads to a fundamentals-based run on the bank. This run is partial and only involves investors who are able to withdraw before the government intervenes. The bank knows this partial run will occur and could prevent it by imposing an initial bail-in large enough to discourage early withdrawals. But doing so is costly because

[3]See "Recommendation of the European Central Bank of 27 March 2020 on dividend distributions during the COVID-19 pandemic," *Official Journal of the European Union*, 2020/C 102 I/01. The European Systemic Risk Board made a similar recommendation on May 27. See Acharya et al. (2016) for a discussion of bank dividend payments in the 2007-9 financial crisis and a model of dividend-related externalities across banks.

a bail-in reduces the bank's bailout dollar-for-dollar. In this way, our model identifies a new channel through which bailouts can increase financial fragility: by raising the bank's cost of using bail-ins to discourage early withdrawals.

Given these problems, we ask whether regulation can improve outcomes. The regulator has the ability to restrict all payments the bank makes to investors, which can be interpreted as limiting dividend payments, imposing withdrawal fees, or writing down the face value of liabilities. However, because the regulator only observes the bank's loss after some withdrawals have been made, it does not know the appropriate level for the initial bail-in. We formulate the regulator's decision as a *delegation problem* in the spirit of Holmström (1977, 1984). The regulator decides to what extent it will require the bank to bail-in the investors who withdraw early and to what extent it will delegate that decision to the bank. The aim of the policy is to increase the bail-in applied in states where the bank will later be bailed out while minimizing the distortion created in states where no bailout occurs. We show that, as long as the bank is bailed out in some states, regulation can improve welfare.

We characterize the optimal policy and show it takes one of two forms. If parameter values are such that a run will never occur, regardless of the bank's choice, the optimal policy is to impose a minimum bail-in. This minimum will bind in states where the bank's loss is large enough for a bailout and also in states where the bank's loss is small or zero. In between, the bank chooses a bail-in above the required minimum, which demonstrates the value of giving it some discretion. For other parameter values, the optimal policy uses the threat of a run by investors to discipline the bank's choice. In these cases, the optimal policy allows the bank to choose from a set of small initial bail-ins or a set of large ones, but not values in between. The policy is calibrated so the bank would suffer a run if it chose a small bail-in following a large loss. The desire to avoid a run leads the bank to self-select into a large bail-in in these states. In cases where a run occurs in the absence of regulation, this approach improves financial stability as well as welfare.

Overall, our results demonstrate the value of regulating bail-ins early in a crisis, rather than waiting for more precise information about banks' losses. Much of the existing policy discussion has focused on tying bail-ins to information that is observed by regulators, either publicly or privately. For example, contingent-convertible bonds (CoCos) can be structured to convert from debt to equity when the book value of a bank's equity falls below some pre-specified level.[4] In our model, the regulator can solve this *ex post* problem by imposing the efficient bail-in on those investors who remain when it observes the bank's loss. There is, however, an earlier period when the regulator knows a problem may exist, but does not yet know how badly the bank is affected. Our results show the value of acting promptly

[4]See Flannery (2014) for a detailed discussion of CoCos and a review of the relevant literature.

to bail-in withdrawals made during this period. Even though this policy tool is blunt, the benefit of increasing the initial bail-in in states where the bank's loss is large outweighs the cost of imposing too large of a bail-in in states where the bank is sound.

Related literature. Wallace (1988, 1990) provided an early analysis of bank bail-ins in a version of the Diamond-Dybvig model with aggregate risk. He showed that when a bank observes withdrawal demand gradually, through a process of sequential service, the efficient allocation has a feature that he called "partial suspension of convertibility" but which in current terminology would be called a bail-in. Subsequent work derived the efficient pattern of bail-ins within an individual bank for different specifications of the environment; see, for example, Green and Lin (2003), Peck and Shell (2003), Ennis and Keister (2009b), and Sultanum (2014). This literature emphasizes that investors *want* their bank to use bail-ins to efficiently allocate risk; there is no need for regulation or supervisory bail-ins in these models. From a policy perspective, this literature broadly supports the type of reforms adopted for money market mutual funds in the U.S. in 2014. In particular, it suggests that if intermediaries are allowed to take bail-in actions such as limiting withdrawals and imposing withdrawal fees, they will do so in times of stress and these actions will promote financial stability. In our setting, in contrast, the anticipation of being bailed out undermines the incentive to use bail-ins, rendering such reforms ineffective and creating a rationale for requiring bail-ins even before policy makers have precise information about a bank's loss.

Our work also relates to an emerging literature that studies the incentive effects of bail-ins and the resulting policy tradeoffs. Bernard et al. (2022) study a game in which a regulator and banks negotiate over the allocation of losses, focusing on how the network structure of interbank linkages affects the credibility of a no-bailout plan. Walther and White (2019) study how a bail-in improves a bank manager's incentive to exert effort by increasing her stake, but risks provoking a run if it leads creditors to infer the bank is in bad shape. Colliard and Gromb (2018) study the negotiation between a bank's shareholders and its creditors over how the losses will be distributed, while Bolton and Oehmke (2019) study the problem of coordinating bail-ins in multinational banks. Overall, this literature focuses on how a regulator should react to the information it receives about about a bank's situation. We focus instead on an earlier stage, when the regulator has limited bank-level data and bank insiders have private information. We show that the regulator should act promptly rather than waiting for bank-specific information to arrive.

2 The model

We base our analysis on a version of the Diamond and Dybvig (1983) model in which policy makers have limited commitment and can provide bailouts, as in Keister (2016).[5] We add private information about the value of the bank's assets to this framework. In this section, we describe the agents, preferences, and technologies that characterize the environment, and we define bail-ins and bailouts within this environment.

2.1 The environment

There are three time periods, labeled $t = 0, 1, 2$, and a single consumption good.

Investors. A continuum of investors, indexed by $i \in [0, 1]$, each have preferences characterized by

$$u(c_1^i + \omega^i c_2^i) = \frac{1}{1-\gamma} \left(c_t^i + \omega^i c_2^i\right)^{1-\gamma}, \tag{1}$$

where c_t^i denotes consumption in period $t \in \{1, 2\}$. The random variable $\omega^i \in \Omega \equiv \{0, 1\}$ is realized at $t = 1$ and is privately observed by the investor. If $\omega^i = 0$, she is *impatient* and values consumption only in period 1, whereas if $\omega^i = 1$, she is *patient*. Each investor will be impatient with a known probability $\pi > 0$, and the fraction of impatient investors will also equal π. As is standard, we assume the coefficient of relative risk aversion γ is strictly greater than 1. Investors are each endowed with one unit of the good at $t = 0$.

The bank. There is a single technology for storing goods across periods; this technology pays a gross return of 1 between periods 0 and 1 and $R > 1$ between periods 1 and 2. As in Diamond and Dybvig (1983), these returns and the idiosyncratic uncertainty about preference types ω^i create an incentive for investors to pool resources in a *bank* to insure against individual liquidity risk. The bank holds the pooled goods in this technology and each investor can contact the bank to withdraw funds at either $t = 1$ or $t = 2$. To simplify the analysis, we begin with the endowment of investors already deposited in the bank.[6]

At $t = 0$, before any decisions are made, the bank experiences a loss whose size is random. Specifically, a fraction λ of the goods held by the bank become worthless, leaving the bank with $1 - \lambda$ units of the good per investor. The loss λ is drawn from a distribution F on the

[5]Limited commitment was introduced into the Diamond-Dybivg framework by Ennis and Keister (2009a, 2010) and has been used to study a range of topics, including how financial fragility is affected by interest rates (Li, 2017), inequality (Mitkov, 2019), asset opacity (Izumi, 2021) and competition (Gao and Reed, 2021; Xiao, 2022).

[6]That is, we do not study what Peck and Shell (2003) call the *pre-deposit game*, in which investors decide whether to pool their resources. We discuss how bail-ins would affect the incentive of investors to deposit in a bank in Section 6.3.

interval $\Lambda \equiv [0, \bar{\lambda}]$. We assume F is continuous, strictly increasing and differentiable, but may place positive probability on $\lambda = 0$. The bank and investors observe the realization of λ before making any decisions.

Investors observe λ and their own preference type, then decide in which period to withdraw. We use $y^i \in \{1, 2\}$ to denote the choice of investor i. Those investors who chose to withdraw in period 1 arrive at the bank one at a time, in a randomly-determined order, and the bank chooses how much consumption each investor receives. Investors are isolated from each other during the withdrawal process and must consume immediately after withdrawing. As in Wallace (1988), this assumption prevents the fiscal authority from being able to tax past withdrawals when new information is revealed.[7]

The fiscal authority. There is a benevolent fiscal authority that has access to goods in period 1 at a marginal cost $\mu > 0$ and the ability to bail out the bank following a loss. After a fraction π of investors have withdrawn, the fiscal authority observes both the realized value of λ and the remaining resources in the bank, then chooses a bailout payment $b \in \mathbb{R}_+$. The fiscal authority's objective is to maximize the sum of investors' utilities minus the cost of the bailout payment. It is unable to commit to a bailout plan in advance; the payment will be chosen as a best response to the situation at hand.

Allocations. An allocation in this environment consists of consumption bundle for each investor and a bailout payment. An allocation is *feasible* if

$$\int_0^1 \left(c_1^i + \frac{c_2^i}{R} \right) di \leq 1 - \lambda + b. \tag{2}$$

In other words, the present value of all consumption can be no larger than the value of the bank's assets plus the bailout payment.[8]

The regulator. In Section 6, we introduce a regulator who can restrict the payments the bank makes to withdrawing investors in period 1. The anticipation of being bailed out distorts the bank's incentives in choosing these payments, which may allow regulation to raise welfare. However, like the fiscal authority, the regulator has limited information; it observes the bank's realized loss only after a fraction π of investors have withdrawn.

[7] The assumption also prevents trading opportunities from undermining the bank's ability to provide liquidity insurance. See Jacklin (1987), Allen and Gale (2004) and Farhi et al. (2009), among others, for studies of how the presence of markets at $t = 1$ limits the amount of risk-sharing that banks provide to depositors.

[8] Because the bailout payment is made after a fraction π of investors have withdrawn, feasibility also requires that payments to these investors total no more than the value of the bank's own assets, $1 - \lambda$. We place assumptions on parameter values below that ensure the bank's choices always satisfy this additional constraint.

2.2 Bail-ins and bailouts

Our interest is in studying how the bank's loss λ is divided between between lower consumption for investors (i.e., bail-ins) and a bailout payment. Measuring bail-ins requires comparing the allocation to some benchmark that investors would receive in normal times. In this subsection, we first define a reference allocation from which bail-ins are measured. We then discuss the particular form an allocation takes in our model and how this form depends on investors' withdrawal behavior.

A reference allocation. Suppose the bank has experienced no loss ($\lambda = 0$). The efficient allocation of resources then gives a common amount c_1 to each impatient investor at $t = 1$ and a common amount c_2 to each patient investor at $t = 2$. These values are chosen to maximize investors' expected utility subject to the feasibility constraint

$$\pi c_1 + (1 - \pi) \frac{c_2}{R} \leq 1. \tag{3}$$

Let (c_1^*, c_2^*) denote the solution to this standard Diamond-Dybvig allocation problem, which satisfies $1 < c_1^* < c_2^* < R$. We consider c_1^* and c_2^* to represent the *face value* of the bank's liabilities to investors who choose to withdraw in periods 1 and 2, respectively. To be clear: the bank in our model can pay withdrawing investors less than these values following a loss and will do so whenever it is in investors' best interests. In this sense, the liabilities defined above are not contractually binding, and deviating from these payments does not involve any cost or inefficiency. The reference amounts (c_1^*, c_2^*) simply provide a benchmark for measuring the portion of the bank's losses that are borne by its investors.

Bail-ins. Following a loss, it may not be feasible for the bank to pay (c_1^*, c_2^*) to its withdrawing investors. In this case, the bank will choose the best feasible allocation of its resources. No information is revealed while the first fraction π of investors withdraw in period 1 and, because investors are risk averse, the bank will choose to give the same amount to each of them. If these investors receive less than the reference amount c_1^*, we say they have been *bailed-in*.[9] It is convenient to measure the size of the bail-in as the percentage "haircut" from the reference allocation, that is, as the solution h to

$$c_1 = (1 - h)\, c_1^*.$$

[9]While some authors apply the term *bail-in* only to losses imposed on certain types of investors (such as long-term debt holders) or in certain situations (such as in resolution), we use the term more broadly to include all losses imposed on a bank's creditors and investors. Our approach aims to capture, in a unified way, a variety of policies and actions observed in reality during financial crises, including restrictions on dividend payments as well as haircuts imposed on depositors, various debt holders, and other creditors.

In period 2, the bank's matured assets will be evenly divided among the remaining investors. Let $\hat{h}$ denote the bail-in applied to these investors, which satisfies

$$c_2 = \left(1 - \hat{h}\right) c_2^*.$$

If a bank has no loss ($\lambda = 0$), it sets $h = \hat{h} = 0$ and implements the reference allocation. If $\lambda > 0$, the bank chooses $(h, \hat{h})$ to maximize investors' expected utility from consumption, subject to feasibility constraints and anticipating the actions of the fiscal authority.

We assume the upper bound on the distribution F satisfies $\bar{\lambda} \leq 1 - \pi c_1^*$. Recall that the fiscal authority makes its bailout decision after a fraction π of investors have withdrawn. This condition thus implies the bank's loss is never so large that it will run out of resources before the bailout decision, regardless of its choice of initial bail-in h.

Bailouts. After a fraction π of investors have withdrawn, the fiscal authority observes the realized value of λ as well as what the bank has left after serving these π withdrawals. It then chooses a bailout payment $b \geq 0$ to maximize the sum of investors' utilities minus the cost μb of the bailout. Note that this decision is made after the bank's initial bail-in h has been applied to the first π investors, and the fiscal authority cannot commit to the bailout policy before these withdrawals are made.

Feasibility. Following a loss λ and bailout b, the bank will have $1 - \lambda + b$ units of the good in period 1. Suppose for the moment that patient investors wait until period 2 to withdraw, so that only a fraction π of investors withdraw at $t = 1$. Then the feasibility constraint in equation (2) can be written in terms of the bail-ins $(h, \hat{h})$ as

$$\pi (1 - h) c_1^* + (1 - \pi) \left(1 - \hat{h}\right) \frac{c_2^*}{R} \leq 1 - \lambda + b.$$

Using equation (3), we can rewrite this constraint as

$$h \pi c_1^* + \hat{h} (1 - \pi) \frac{c_2^*}{R} + b \geq \lambda. \tag{4}$$

The first two terms of the left-hand side measure the period-1 value of the bail-ins: an amount πc_1^* of the bank's liabilities is bailed in at rate h in period 1, while the amount $(1 - \pi) c_2^*$ that will be bailed in at rate $\hat{h}$ in period 2 is discounted by the return R. Feasibility requires the sum of these bail-ins plus the bailout payment b to cover the bank's loss λ.

Bank runs and resolution. Throughout our analysis, we assume patient investors wait until $t = 2$ unless withdrawing early is a strictly dominant strategy. In other words, we do not focus on the type of self-fulfilling bank runs studied by Diamond and Dybvig (1983) and many others. There may, however, be situations in which patient investors receive more by withdrawing early regardless of the actions of others. In such cases, a bank run is inevitable.[10]

If investors continue to arrive at the bank in period 1 after π withdrawals have been made, the bank is placed into *resolution*. We assume this process stops the run, so subsequent withdrawals at $t = 1$ are made only by impatient investors. In addition, the fiscal authority observes the fraction of remaining investors who are impatient and conditions the bailout payment (if any) on this information. When a bank is in resolution, the fiscal authority dictates the bail-ins applied to all remaining investors, which implies that the bank's remaining resources will be allocated efficiently among its remaining investors.

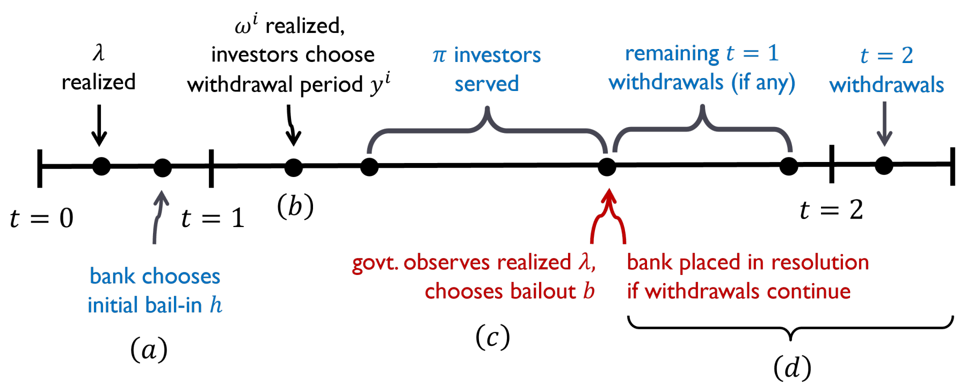


Figure 1: Timeline

The sequence of events is summarized in Figure 1, where items in black represent moves by nature and individual investors, items in blue represent the actions of the bank, and items in red correspond to the actions of the fiscal authority.

2.3 Discussion

Cost of public funds. The parameter μ measures the fiscal authority's marginal cost of resources in the crisis state. One can interpret μ as reflecting the marginal value of public goods and services that are foregone when the funds are used instead for a bailout, as in Keister (2016), or as the value of private consumption foregone when additional taxes are raised. The key assumption we make here is that this marginal value is fixed, independent of the size of the bailout payment. The assumption would be satisfied, for example, if utility is linear in the public good or if the size of the equilibrium bailout payment is small relative

[10]In focusing on bank runs that are driven by the "fundamentals" of the withdrawal game, we follow Chari and Jagannathan (1988) and Allen and Gale (1998), among others.

to the overall government budget constraint. We assume the marginal cost μ satisfies

$$\mu > u'(c_1^*) \equiv \underline{\mu}, \tag{5}$$

which ensures the fiscal authority would not want to bail out the bank when there is no loss.

Number of banks. Our model has a single bank and, therefore, a single realized loss λ. However, the analysis would be unchanged if there are many banks that receive idiosyncratic draws from the loss distribution F. The loss in each bank would then be divided between a bail-in of that bank's investors and a bailout from the fiscal authority. As long as the fiscal authority's marginal cost of funds is unchanged, the outcome at one bank does not effect the incentives of other banks or their investors.[11] We return to the multiple-bank interpretation of the model in the context of regulatory policy in Section 6.3.

Distribution of loss. We take the distribution F of the bank's loss as exogenous to our model. In this sense, our focus is on how a bank's loss is allocated and not on the determinants of the loss or *ex ante* moral hazard issues.[12] Our approach is particularly relevant for studying the effects of unexpected economic shocks originating outside of the financial sector. However, the effects we highlight in our analysis will be present any time banks face a significant loss, regardless of the underlying cause.

Bank's objective. The bank chooses how much to pay withdrawing investors with the objective of maximizing their expected utility from consumption. The cost of funds for any bailout payment is external to the bank (and its investors). In an economy composed of many banks, for example, the cost of bailing out any one bank will fall largely on investors in other banks and on other sectors of the economy. This external effect is what creates a potential role for regulation, as we discuss in Section 6.

No commitment. Our assumption that the fiscal authority cannot commit to not bail out the bank follows a large literature on bailouts and seems well aligned with historical experience.[13] This assumption prevents the fiscal authority from using bailout policy to reward or punish particular choices of initial bail-in by the bank. It can be interpreted as capturing elements of "too big to fail" in the sense that the fiscal authority finds the costs

[11] If the fiscal authority's marginal cost of funds instead changes depending on the size of the aggregate bailout, strategic interactions emerge as the bail-in chosen by one bank affects the bailouts received by others. See Keister and Mitkov (2020) for a version of the model in which those effects are present.

[12] A large literature has studied how government guarantees, both explicit and implicit, affect the riskiness of banks' assets and, therefore, the probability of a crisis state. Kareken and Wallace (1978) is one classic reference. Acharya and Yorulmazer (2007) study how the anticipation of intervention affects the correlation of banks' asset choices and, therefore, the distribution F of losses across banks in a crisis.

[13] See, for example, Farhi and Tirole (2012), Chari and Kehoe (2016), Bianchi (2016), Keister (2016), Nosal and Ordonez (2016), and Dávila and Walther (2020).

associated with a large loss λ too high to leave unaddressed. Precisely how large λ must be to elicit a bailout is endogenous in our model and depends on the bank's choice of initial bail-in h as well as on the cost of public funds μ. This interaction between the choice of bail-in and the subsequent bailout is at the heart of our analysis.

Delayed intervention. The fiscal authority observes the bank's loss with a delay, reflecting the idea that banks are likely to have more information about their own situation than is available to regulators in the early stages of a crisis. In direct terms, this assumption aims to capture the time required to carry out detailed examinations and to verify the information that forms the basis for supervisory action. More broadly, the delay in the model can also represent a variety of practical and political concerns that make policy makers slow to react to an incipient crisis. (See, for example, Kroszner and Strahan (1996), Brown and Dinc (2005), Iyer et al. (2016), and Nosal and Ordonez (2016).) The key point for our analysis is simply that some investors are able to withdraw from a bank facing losses before decisions about bailouts and bank resolution are implemented.

Informed investors. Investors are informed about the value of the bank's assets at the beginning of period 1. Because of sequential service, however, only a fraction π of them can act on this information before the fiscal authority intervenes. In effect, the private information is only relevant for this group of investors, which we interpret as representing insiders to the bank.[14] Assuming that all investors are informed and, hence, face the same decision problem helps simplify the presentation of our model. The important point, however, is that some investors are able to act on their information before intervention occurs.

Resolution. We assume the bank is placed in resolution as soon as it becomes apparent to the fiscal authority that a run is underway. Once in resolution, the bank's available resources – including any bailout it receives – are allocated efficiently among its remaining investors. There are a variety of ways to implement this resolution, all of which lead to the same outcome in our model. One could, for example, think of a court system intervening to verify investors' preference types, as discussed in Ennis and Keister (2009a). Alternatively, one could allow investors to write a "living will" that specifies how their bank will be operated following a run and intervention. Because there are no further bailouts at this point, investors' incentives are no longer distorted and the allocation of the bank's remaining resources will be the same regardless of who chooses the payments. Our approach of having the fiscal authority dictate all remaining payments serves only to simplify the notation.

[14] Several studies have highlighted the importance of withdrawals by bank insiders in the period before regulatory actions and/or bank failure occur. See, for example, Acharya et al. (2011), Henderson et al. (2015), and Iyer et al. (2016).

3 A planner's problem

In this section, we derive the combination of bail-ins and bailouts that would be chosen by a benevolent planner who controls the operations of the bank, investors' withdrawal decisions, and the actions of the fiscal authority. This planner observes all of the information available to investors, including the bank's loss and each investor's preference type, and wants to maximize investors' expected utility minus the cost of bailouts.

The planner will direct all impatient investors to withdraw at $t = 1$ and all patient investors to withdraw at $t = 2$. Given the realized loss λ, the planner will choose the bail-ins $(h, \hat{h})$ and the bailout b to maximize

$$\pi u\left(\left(1-h\right)c_1^*\right) + (1-\pi)u\left((1-\hat{h})c_2^*\right) - \mu b$$

subject to the feasibility constraint in equation (4) and the non-negativity restrictions

$$h \in [0,1], \quad \hat{h} \in [0,1], \quad \text{and} \quad b \geq 0.$$

Let $\{h^*, \hat{h}^*, b^*\}$ denote the solution to this problem, which we call the *efficient plan*. Our first result shows that the planner will impose the same bail-in on all investors.

Proposition 1. *The efficient plan satisfies $h^* = \hat{h}^*$ for all $\lambda \in \Lambda$.*

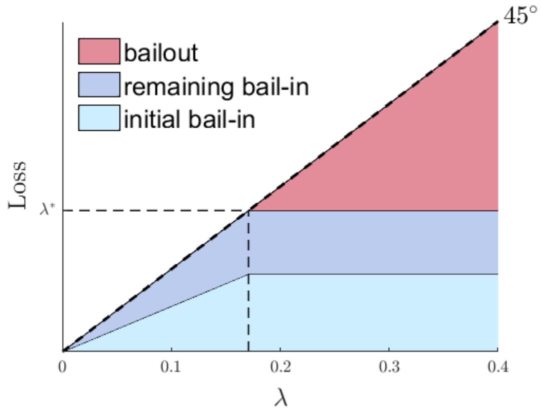
Proofs of all propositions are provided in an online appendix. The result in Proposition 1 relies on the form of the utility function in equation (1), which implies investors' expected-utility preferences are homothetic. The efficient levels of consumption for impatient and patient investors thus scale down in proportion when the bank experiences a loss.

Using Proposition 1 together with the resource constraint for the reference allocation in equation (3), we can rewrite the feasibility constraint (4) in a particularly simple form:

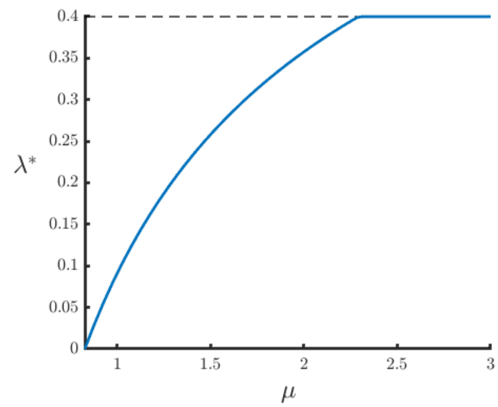
$$h + b \geq \lambda. \tag{6}$$

In other words, the loss λ must be covered by a combination of bail-ins h of the bank's investors and bailouts b by the public sector. Our next result characterizes the planner's choice of this division. Define λ^* to be the loss such that the marginal utility of impatient investors when there is no bailout exactly equals the marginal cost of public funds, that is,

$$u'\left((1-\lambda^*)c_1^*\right) = \mu \qquad \text{or} \qquad \lambda^* = 1 - \frac{1}{\mu^{\frac{1}{\gamma}} c_1^*}. \tag{7}$$



(a) Allocation of losses



(b) Bailout cutoff λ^* as μ varies

Figure 2: The efficient allocation

Note that our assumption in equation (5) implies $\lambda^* > 0$. We then have the following result.

Proposition 2. *The efficient plan* $\left(h^*, \hat{h}^*, b^*\right)$ *sets*

$$h^* = \hat{h}^* = \left\{ \begin{array}{c} \lambda \\ \lambda^* \end{array} \right\} \quad \textit{and} \quad b^* = \left\{ \begin{array}{c} 0 \\ \lambda - \lambda^* \end{array} \right\} \quad \textit{as} \quad \lambda \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} \lambda^*.$$

The efficient plan is characterized by a maximum bail-in of λ^*. When the bank's loss is smaller than this maximum, the efficient bail-in h^* equals the total loss and no bailout is received. When the bank's loss is larger than λ^*, the maximum bail-in is applied and the remaining loss is covered by a bailout. In this way, the planner uses public resources to provide insurance against large losses, but not against smaller ones.

Panel (a) of Figure 2 illustrates this result.[15] The graph shows, in each state λ, how the loss is divided between a bail-in of the first π investors to withdraw (bottom region, light-blue), a bail-in of the remaining $(1-\pi)$ investors (middle region, darker blue), and a bailout (top region, red). Note that the relative sizes of the first two regions are the same for all λ, in line with Proposition 1. When the loss is smaller than λ^*, these bail-ins cover the entire loss and there is no bailout. When the loss is larger than λ^*, the bail-ins take the maximum value and the bailout covers the remaining loss.

The cutoff value λ^* depends critically on the marginal cost of resources μ for the fiscal authority, as show in equation (7) and illustrated in panel (b) of the figure. When μ is equal to the lower bound $\underline{\mu}$ in equation (5), we have $\lambda^* = 0$ and the bank receives a bailout even

[15] All of the numerical examples in the paper use $\gamma = 2$, $\pi = 1/2$ and $R = 1.5$. The distribution F places measure $1/2$ on $\lambda = 0$ and the remaining measure is uniformly distributed on $\Lambda = [0, 0.4]$. Panel (a) of Figure 2 uses $\mu = 1$.

if it experiences a very small loss. As the marginal value of public resources increases, the optimal bailout policy becomes less generous and the cutoff λ^* rises. When μ is large enough, the cutoff equals the upper bound of the loss distribution $\bar{\lambda}$, meaning the bank is not bailed out even if it experiences the largest possible loss.

4 Decentralized allocations

We now return to the decentralized economy, where the initial bail-in is chosen by the bank, withdrawal decisions are made by investors, and the bailout is chosen by the fiscal authority. In this section, we derive the allocation that results from each possible choice of the initial bail-in h by working backward through the timeline in Figure 1. We start with the allocation of consumption to the remaining investors at point (d), then move to the bailout decision at point (c) and investors' withdrawal choices at point (b). Together, these actions generate a mapping from the initial bail-in h to the resulting allocation. In Section 5, we use this mapping to characterize the bank's optimal choice of h at point (a) in the timeline.

4.1 Remaining withdrawals

At point (d) in the timeline, a fraction π of investors have already withdrawn and consumed. Let h denote the bail-in that was imposed on these initial withdrawals, and let b denote the bailout payment that was made by the fiscal authority at point (c). Then the resources the bank has available for each of its $(1-\pi)$ remaining investors at point (d) are

$$\psi(h,b) \equiv \frac{1-\lambda-\pi\,(1-h)\,c_1^*+b}{1-\pi}. \tag{8}$$

The composition of these remaining investors between patient and impatient types depends on the withdrawal decisions made at point (b). Let $y \in \{1,2\}$ denote the action of patient investors, which is necessarily symmetric because we assume they run only when doing so is a strictly dominant strategy. If $y=2$, meaning patient investors chose to wait, the first π withdrawals were made by impatient investors and all remaining investors at point (d) are patient. Each of these investors will receive $R\psi$ at $t=2$. If $y=1$, the bank experienced a partial run at point (c) and the remaining investors are a mixture of patient and impatient types. In this case, the bank is placed into resolution and the fiscal authority allocates the remaining resources to maximize the sum of these investors' utilities. It is straightforward to show the solution to this allocation problem gives ψc_1^* to each remaining impatient investor at $t = 1$ and ψc_2^* to each remaining patient investor at $t = 2$.[16] Converting these consumption values into haircuts shows the bail-ins faced by the remaining investors to be

$$\hat{h}(h, y, b) = \left\{ \begin{array}{c} 1 - \frac{R}{c_2^*}\psi(h, b) \\ 1 - \psi(h, b) \end{array} \right\} \quad \text{as} \quad y = \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\}. \tag{9}$$

Given an initial bail-in h, withdrawal behavior y, and bailout b, equations (8) and (9) show how the resources remaining in the bank at point (d) in the timeline will be allocated. Let $V(\psi, y)$ denote the average utility of the bank's remaining investors, that is,

$$V(\psi, y) \equiv \left\{ \begin{array}{c} u(R\psi) \\ \pi u(\psi c_1^*) + (1-\pi)u(\psi c_2^*) \end{array} \right\} \quad \text{as} \quad \left\{ \begin{array}{c} y = 2 \\ y = 1 \end{array} \right\}. \tag{10}$$

4.2 Bailout payment

We next consider point (c) in the timeline, where the fiscal authority makes its bailout decision. It recognizes the bank's remaining resources will be utilized as derived above and, given h and y, chooses the bailout b to maximize

$$(1-\pi)V(\psi(h, b),\, y) - \mu b$$

subject to the non-negativity constraint $b \geq 0$. The first-order condition for this problem is

$$V_1(\psi(h, b), y) \leq \mu \quad \text{with equality if } b > 0. \tag{11}$$

If this condition holds with equality, it is straightforward to show that the bail-in imposed on the remaining investors, $\hat{h}$, is equal to λ^*. In other words, if the bank is bailed out, b will be chosen so the bank's remaining investors are bailed in at the same rate that the planner would chose when the bank is bailed out.To determine whether or not the bank is bailed out, we compare λ^* to the bail-in the remaining investors would face if there were no bailout, denoted $\hat{h}_{NB}$. Substituting equation (8) into equation (9) and setting $b = 0$, we have

$$\hat{h}_{NB}(h, y) = \left\{ \begin{array}{c} 1 - \left(\frac{R}{c_2^*}\right)\frac{1-\lambda-\pi(1-h)c_1^*}{1-\pi} \\ 1 - \frac{1-\lambda-\pi(1-h)c_1^*}{1-\pi} \end{array} \right\} \quad \text{as} \quad y = \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\}. \tag{12}$$

[16] This result is similar in spirit to Proposition 1 and also relies on the form of the utility function in equation (1). If preferences did not have the constant-relative-risk aversion form, the analysis would be similar but the efficient allocation of bank's remaining resources would apply different bail-ins to the remaining patient and impatient investors following a run.

The following result uses $\hat{h}_{NB}$ to characterize the fiscal authority's choice of bailout payment and the bail-in $\hat{h}$ of the bank's remaining investors.

Proposition 3. *Given h and y, the bail-in of remaining investors $\hat{h}$ and bailout b satisfy:*

(i) if $\hat{h}_{NB}(h, y) \leq \lambda^*$, then $\hat{h}(h, y) = \hat{h}_{NB}(h, y)$ and $b(h, y) = 0$

(ii) if $\hat{h}_{NB}(h, y) > \lambda^*$, then $\hat{h}(h, y) = \lambda^*$ and

$$b(h, y) = \left\{ \begin{array}{c} (1 - \pi c_1^*) \left(\hat{h}_{NB}(h, 2) - \lambda^* \right) \\ (1 - \pi) \left(\hat{h}_{NB}(h, 1) - \lambda^* \right) \end{array} \right\} \text{ as } y = \left\{ \begin{array}{c} 2 \\ 1 \end{array} \right\}. \tag{13}$$

If the bail-in investors would face with no bailout is less than λ^*, the fiscal authority will not bail out the bank. If it is larger, the fiscal authority will provide a bailout than lowers the bail-in of remaining investors to λ^*. The size of the bailout required to achieve this outcome depends not only on the bank's remaining resources, but also on the composition of its remaining depositors between patient and impatient types, as shown in equation (13).

Overall, Proposition 3 shows that, after the first π withdrawals have occurred, the allocation of the bank's remaining loss takes the same general form as the solution to the planner's problem characterized in Proposition 2. In particular, the bailout policy ensures the remaining investors never experience a bail-in larger than λ^*, and the bank is not bailed out when a smaller bail-in is required to cover the losses. Looking ahead, however, Proposition 3 also illustrates how the equilibrium bailout policy will tend to distort the bank's choice of initial bail-in h. A smaller initial bail-in will leave the bank with fewer resources, increasing $\hat{h}_{NB}$. Equation (13) shows the fiscal authority will respond with a larger bailout b, effectively rewarding the bank for its choice. This incentive distortion creates a wedge between the equilibrium outcome and the efficient plan.[17] Before discussing this wedge in detail, we need to analyze investors' withdrawal choices.

4.3 Withdrawal choices

At point (b) in the timeline, investors choose when to withdraw. Impatient investors only value consumption in period 1 and, therefore, always choose to withdraw early. A patient investor chooses to withdraw in whichever period she would receive more. She anticipates that the bailout at point (c) and the subsequent bail-in of remaining investors at point (d) will be as described above. Using the function $\hat{h}$ defined in equation (9), waiting to withdraw

[17] Note that this wedge would not arise if the fiscal authority could commit in advance to a bailout schedule that depends on the realized loss λ but not on the bank's choice of initial bail-in h. The inability to commit to such a plan is why bailouts distort incentives.

is a best response for a patient investor if and only if

$$(1-h)\, c_1^* \leq (1-\hat{h})c_2^*.$$

The bail-in $\hat{h}$ on the right-hand side of this condition depends on whether the bank is bailed out, which itself depends on investors' withdrawal behavior as shown in Proposition 3. We can determine this withdrawal behavior in two steps. First, suppose $\hat{h}_{NB}(h,2) > \lambda^*$, meaning the bank would be bailed out if there is no run. Equation (12) shows that $\hat{h}_{NB}$ becomes larger if a run occurs, so the bank would be bailed out following a run as well. Either way, Proposition 3 shows that a patient investor who waits will receive $(1-\lambda^*)c_2^*$. Notice this value does not depend on the withdrawal behavior of other investors. The strategic complementarity that usually generates multiple equilibria in the Diamond-Dybvig framework is absent here because the bailout makes up for the additional losses created by a run.

If $\hat{h}_{NB}(h,2) \leq \lambda^*$, the bank will not be bailed out if there is no run. In this case, the standard strategic complementarity is present and the withdrawal game may have multiple equilibria. In particular, equation (12) shows how early withdrawals by other patient investors increase the bail-in $\hat{h}_{NB}$ of the remaining investors and thus increase the incentive to withdraw early. Given that this type of bank run has been extensively studied, we do not focus on it here. Instead, we assume that patient investors withdraw early only if doing so is a strictly dominant strategy. Patient investors' withdrawal behavior thus given by

$$y(h) = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\} \quad \text{as} \quad (1-h)c_1^* \left\{ \begin{matrix} \leq \\ > \end{matrix} \right\} \Big(1 - \min\{\hat{h}_{NB}\,(h,2)\,, \lambda^*\}\Big)\, c_2^*. \tag{14}$$

The next proposition shows that both investors' withdrawal behavior and the bailout payment are monotone functions of the bank's initial bail-in.

Proposition 4. *The function $y(h)$ in equation (14) is weakly increasing. The composite function $b\,(h, y(h))$ defined by equations (13) and (14) is decreasing in h and is strictly decreasing whenever $b(h, y(h)) > 0$.*

These results are intuitive. First, a larger bail-in makes withdrawing early less attractive and thus decreases the incentive for patient investors to run. Second, a larger bail-in also leaves the bank with more resources when the bailout decision is made, which leads the fiscal authority to provide a smaller bailout.

4.4 Payoffs

To summarize the analysis in this section, it is helpful to look back at the timeline in Figure 1. Given the realized loss λ and the bank's choice of initial bail-in h at point (a) in the timeline, equation (14) uniquely determines investors' withdrawal behavior at point (b). Given this withdrawal behavior, Proposition 3 determines the bailout chosen by the fiscal authority at point (c), and then equations (8) and (10) determine how the bank's remaining resources are allocated at point (d). In other words, we now have a mapping from the bank's choice of initial bail-in h to the resulting allocation of consumption and the associated bailout. We can use this mapping to write investors' indirect utility as a function of the bail-in h as

$$W_B\left(h;\lambda\right) = \pi u\big((1-h)c_1^*\big) + (1-\pi)V\Big(\psi\big(h, b[h, y(h)]\big), y(h)\Big). \tag{15}$$

This expression illustrates the four ways in which the choice of initial bail-in h affects investors' payoffs. The first term is straightforward: a larger bail-in directly reduces the consumption of the first π investors to withdraw. For the remaining $1-\pi$ investors, the bail-in increases the bank's remaining resources before the bailout decision is made, but may decrease the bailout payment. It also discourages early withdrawals. We include λ as an argument of the function W_B to emphasize that all of these relationships depend on the size of the bank's loss.

The relationship between h and aggregate welfare, including the cost of the bailout payment, is given by

$$W_R\left(h;\lambda\right) = W_B\left(h;\lambda\right) - \mu b\big(h, y\left(h\right)\big).$$

The subscript R indicates that this expression will be the objective function for the regulator in Section 6. Before discussing regulation, however, we analyze how the incentives represented in equation (15) determine the bank's choice of initial bail-in h.

5 The initial bail-in

In this section, we characterize the initial bail-in h the bank chooses in the absence of regulation. We also derive the resulting bailout payment and withdrawal decisions of investors. In broad terms, we show that the initial bail-in is too small, the bailout is too large, and these distortions sometimes causes a run on the bank.

5.1 The incentive to bail in

We first discuss the bank's bail-in incentives *holding fixed* whether or not the bank is bailed out. If the bank anticipates no bailout, regardless of its initial bail-in, it is straightforward to show that the optimal choice is $h = \lambda$. Substituting this choice into equations (8) and (9) when $b = 0$ yields $\hat{h} = \lambda$ as well, meaning the bank's loss is shared evenly by all investors. In other words, in the absence of bailouts, the bank would choose the same bail-ins as in the efficient plan (see Propositions 1 and 2).

If the bank anticipates being bailed out, in contrast, Proposition 3 shows that the bail-in $\hat{h}$ of its remaining investors will equal λ^* regardless of the initial bail-in h. Holding the withdrawal decisions of investors fixed, therefore, the bank will want to set the smallest bail-in possible, $h = 0$. In this way, bailouts distort the bank's incentive to bail in. Why impose any loss on these first π investors if doing so reduces the bailout the bank will receive dollar-for-dollar?

There is an important exception to this logic: in some situations, setting $h = 0$ will lead to a run on the bank. The bank can prevent this run by setting its initial bail-in appropriately. Equation (14) shows that investors will not run when the bank is bailed out as long as $(1 - \lambda^*)c_2^* \geq (1 - h)\, c_1^*$. Let $\underline{h}$ denote the smallest initial bail-in h that will prevent a run, that is,

$$\underline{h} \equiv max \left\{ 1 - (1 - \lambda^*)\frac{c_2^*}{c_1^*}, 0 \right\}. \tag{16}$$

We show below that, for some parameter values, the bank will set $h = \underline{h} > 0$. In these cases, the desire to avoid a run partially offsets the incentive distortion created by bailouts. Notice, however, that $c_2^* > c_1^*$ implies $\underline{h} < \lambda^*$. Even in these cases, the bank's choice of initial bail-in is still strictly smaller than in the efficient plan.

This discussion establishes that the initial bail-in will take one of three values when there is no regulation, λ, $\underline{h}$, or 0, and the bank's optimal choice depends on whether it anticipates being bailed out. In many cases, however, whether the bank is bailed out depends on the bail-in it chooses, since a larger bail-in leaves the bank in better condition. The bank then faces a tradeoff. Setting a low initial bail-in (0 or $\underline{h}$) leads to a bailout, which increases the bank's total resources. However, the low bail-in allocates these resources less efficiently across investors than a larger bail-in would. Whether the bank is bailed out thus depends on which of these two forces dominates, which in turn depends on the generosity of the bailout policy, as we show below.

5.2 Optimal choice of bail-in

We divide the analysis of the bank's choice of h into three cases depending on the government's marginal cost of funds μ, which determines the generosity of the bailout policy.

Low cost of funds. If the government's cost of funds is low enough, $\underline{h}$ as defined in equation (16) is zero. This case obtains when

$$1 - \lambda^* \geq \frac{c_1^*}{c_2^*} = R^{-\frac{1}{\gamma}}$$

or, using the definition of λ^* in equation (7), when

$$\mu \leq R\left(c_1^*\right)^{-\gamma} \equiv \mu_1.$$

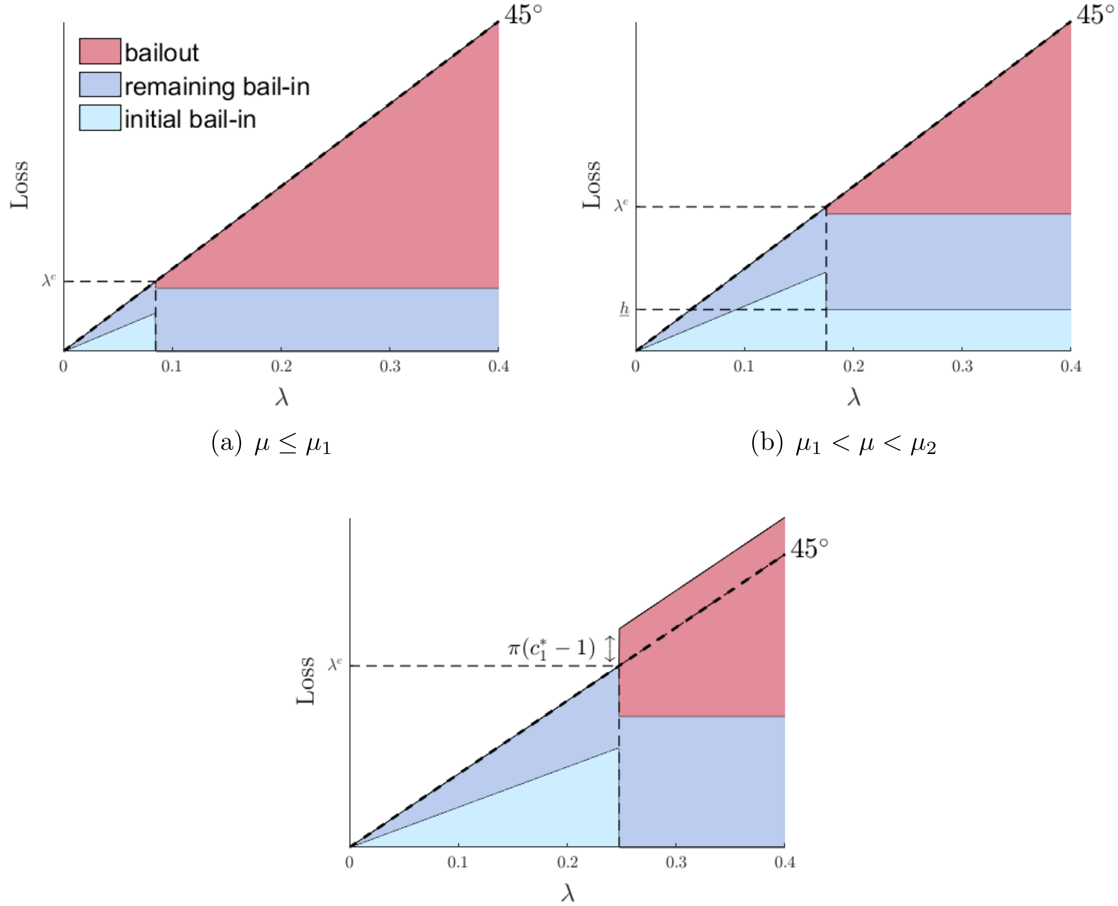
If the bank anticipates being bailed out in this case, it can set $h = 0$ without provoking a run. Our next result characterizes the outcome for this case. We use h^e to denote the bank's optimal choice of initial bail-in; y^e and b^e denote the resulting withdrawal behavior and bailout payment, respectively.

Proposition 5. *There exists $\lambda_1^e < \lambda^*$ such that, when $\mu \leq \mu_1$, the bank is bailed out if and only if $\lambda > \lambda_1^e$. In this region, the bank sets $h^e = 0$, patient investors do not run ($y^e = 2$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + \lambda^* \pi c_1^* \quad > \ b^*.$$

This proposition shows bailouts are more frequent ($\lambda_1^e < \lambda^*$) and larger ($b^e > b^*$) than in the efficient allocation. In particular, b^e equals the efficient bailout b^* ($= \lambda - \lambda^*$) plus a term that represents the bail-in λ^* the planner would have applied to the πc_1^* funds that have already been withdrawn. In other words, the loss that was not imposed on these early withdrawals ends up falling entirely on the public sector, which leads the bank to set $h^e = 0$.

Panel (a) of Figure 3 illustrates the results by depicting the allocation of the bank's loss for the same parameter values as panel (a) of Figure 2, which satisfy $\mu < \mu_1$. Comparing the two figures shows that the bailout region is larger in the decentralized allocation ($\lambda_1^e < \lambda^*$). In addition, there is no light-blue region when $\lambda > \lambda_1^e$; the bank sets $h^e = 0$ and all of the loss falls on the remaining investors and the public sector in this region. As a result, the size of the bailout payment (the red region) is larger for any $\lambda > \lambda^e$. For $\lambda < \lambda_1^e$, there is no bailout and the bail-ins divide the loss proportionally between impatient and patient investors, as in the efficient plan.



(a) $\mu \leq \mu_1$

(b) $\mu_1 < \mu < \mu_2$

(c) $\mu > \mu_2$

Figure 3: Decentralized allocation of losses

Moderate cost of funds. When μ is greater than μ_1, $\underline{h}$ as defined in equation (16) is strictly positive and the bank will experience a run if it sets $h = 0$ and is bailed out. The bank can prevent a run, but doing so is costly because a bail-in of period 1 withdrawals is needed to induce patient investors to wait. Our next result shows that, when the government's cost of funds is not too high, the bank is willing to pay this cost.

Proposition 6. *There exist $\mu_2 > \mu_1$ and $\lambda_2^e < \lambda^*$ such that, when $\mu_1 < \mu < \mu_2$, the bank is bailed out if and only if $\lambda > \lambda_2^e$. In this case, the bank sets $h^e = \underline{h} > 0$, patient investors do not run ($y^e = 2$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + (\lambda^* - \underline{h})\, \pi c_1^* \quad > \ b^*.$$

In this region, the bank's desire to avoid a run partially offsets the moral hazard problem created by bailouts. The bailout b^e again equals the planner's bailout b^* plus a term that represents, for the πc_1^* funds that have already been withdrawn, the difference between the bail-in the planner would have applied (λ^*) and the bail-in the bank did apply ($\underline{h}$). This expression shows how the bank's response to the threat of a run decreases the burden that falls on the public sector. The bank's incentive in choosing h^e is still distorted, of course, and $\underline{h} < \lambda^*$ implies the equilibrium bailout is still inefficiently large.

Panel (b) of Figure 3 illustrates the implications. Unlike in panel (a), the initial bail-in h is now positive for all $\lambda > 0$ (the light-blue region), which decreases the size of the bailout payment (the red region). The higher cost of public funds also makes bailouts less frequent (the threshold λ^e is higher). Bear in mind, however, that a larger value of μ also increases the efficient bailout cutoff λ^*, as shown in equation (7) and illustrated in panel (a) of Figure 4 below. Overall, Proposition 6 shows that bailouts are again more frequent and larger than in the efficient allocation.

High cost of funds. The final case is where the government's cost of funds is higher than μ_2. In this case, the bail-in λ^* imposed on remaining investors following a bailout will be relatively large. Preventing a run would require the bank to set a large initial bail-in $\underline{h}$ as well. Our next result shows that the bank finds preventing a run too costly in this case and instead sets its initial bail-in to zero.

Proposition 7. *There exists $\lambda_3^e < \lambda^*$ such that, when $\mu > \mu_2$, the bank is bailed out if and only if $\lambda > \lambda_3^e$. In this case, the bank sets $h^e = 0$, patient investors run ($y^e = 1$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + \lambda^* \pi c_1^* + (1 - \lambda^*)\pi(c_1^* - 1).$$

As in the previous two cases, bailouts are more frequent and larger than in the efficient allocation. The new feature is that a partial bank run occurs, which creates a further misallocation of resources. The bank could prevent this run by setting the initial bail-in high enough. However, when $\underline{h}$ is very large, the cost of preventing a run outweighs the benefit. The bank allows the run to occur, recognizing that some of the additional losses will be recovered through a larger bailout. The proposition shows that b^e now equals the efficient bailout b^* plus two additional terms. The first of these terms represents the loss that was not imposed on the first π investors, as in Proposition 5. The final term represents the portion of the additional loss caused by the run that falls on the public sector. Panel (c) of Figure 3 shows the allocation of the bank's loss in this third case. Above the bailout cutoff λ_3^e, the bank sets $h^e = 0$ and there is no light blue region. In addition, the sum of the bail-in of remaining investors (medium blue) and the bailout (red) now exceed the loss λ because they must also make up for the misallocation of resources created by the run.

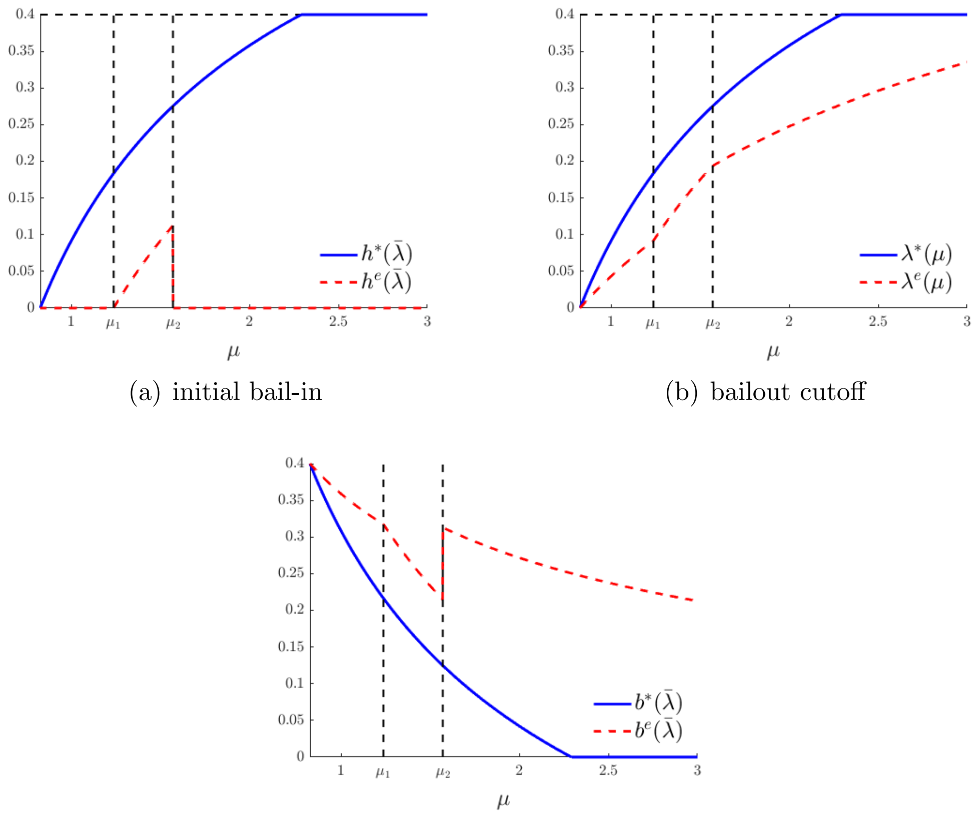
As we continue to increase μ, the bailout cutoff λ^e eventually crosses the upper bound of the loss distribution, $\bar{\lambda}$. Beyond this point, the cost of public funds is so high that the bank is not bailed out in any state and the resulting allocation is always efficient. For the remainder of the analysis, we assume the bank is bailed out in some states, so an incentive distortion arises. This condition is satisfied as long as μ is not too far above μ_2.

5.3 Summary of results

While Propositions 5 - 7 differ in their details, a clear pattern emerges. In all three cases, the initial bail-in is inefficiently small when the bank is bailed out, and bailouts are more frequent and larger than in the efficient allocation. Figure 4 illustrates the inefficiency that arises along each of these margins. Panel (a) compares the decentralized and efficient initial bail-ins h as the cost of public funds μ varies, focusing on the largest possible loss, $\bar{\lambda}$. In line with Propositions 5 - 7, the decentralized bail-in is zero when the cost of public funds is either low or high, but positive in the intermediate range where the bank sets $h^e = \underline{h}$ to prevent a run. Even in this region, the figure shows the decentralized bail-in is always smaller than the planner's choice. Panel (b) plots the efficient and decentralized bailout cutoffs. Both cutoffs are increasing in μ, but the decentralized cutoff λ^e is always smaller. For some values of μ, the efficient cutoff lies at the upper bound of the distribution of λ, meaning the planner would never bail the bank out, but a bailout still occurs in the decentralized economy when the loss is large enough. Finally, panel (c) compares the efficient and decentralized bailouts, again assuming bank has suffered the largest possible loss, $\bar{\lambda}$. The figure shows that both bailout payments are decreasing in the cost of public funds, but the decentralized bailout b^e is always larger. The decentralized bailout b^e jumps up at μ_2 because of the change in withdrawal behavior: a run by patient investors leaves the bank in worse condition, which the fiscal authority responds to with a larger bailout payment.

5.4 Discussion

Source of fragility. Many papers follow the original Diamond-Dybvig approach of assuming a bank must pay withdrawing depositors at face value even after a loss has occurred or it becomes clear a run is underway. (See, for example, Allen and Gale, 1998, Cooper and Ross, 1998, and Goldstein and Pauzner, 2005.) This approach generates a bank run equilibrium, but it is at odds with recent reforms that increase intermediaries' ability to impose losses on creditors. In the absence of bailouts, investors would want their bank to use these tools to prevent runs and efficiently allocate its losses. Other papers allow bail-ins and instead generate fragility by using information structures that make the bank to slow to identify a



(a) initial bail-in

(b) bailout cutoff

(c) bailout payment

Figure 4: Inefficiency of the decentralized allocation

run. For example, investors may condition their withdrawal decisions on a sunspot variable that is unobserved by the bank (as in Peck and Shell, 2003, Ennis and Keister, 2010, and others). Bank runs may be prevented in these models if the bank can detect a run quickly enough, perhaps using an indirect mechanism (as in Andolfatto et al., 2017), or can credibly reveal information about withdrawals as they occur (as in Huang, 2023).

Our model offers a different view of the underlying cause of financial fragility. The contracting and information frictions used in the existing literature are absent; the bank can freely choose the initial bail-in h and perfectly forecasts equilibrium withdrawal behavior. The bank has the usual incentive to avoid a run, which misallocates resources and lowers investors' welfare. However, preventing a run may be costly because the initial bail-in needed to deter early withdrawals would decrease the bank's subsequent bailout. This tradeoff generates an alternative theory of fragility: it results from the bank's anticipation of being bailed out by a fiscal authority with limited commitment, which makes preventing a run privately costly. This theory has novel implications for regulation, which we explore in Section 6.

Bailouts vs. fire sales. Other factors may also distort a bank's bail-in incentives, such as a fire-sale externality that makes the social cost of liquidating investment at $t = 1$ higher than the private cost. However, such factors would not generate the type of fragility we identify here. A fire sale externality would increase the social benefit of bailing-in but would not lead the bank to decrease its initial bail-in relative to a benchmark case with no distortions. A bailout, in contrast, undermines the private incentive to bail in relative to this benchmark. In other words, bailouts make preventing a run costly for the bank in a way that a fire sale externality would not.

Disciplining effect of runs. Our model also shows, however, that bailouts do not completely undermine a bank's incentive to use bail-ins. In some situations, the threat of a run leads the bank to bail-in its investors, even though doing so is costly. This disciplining role of bank runs is similar in spirit to Calomiris and Kahn (1991) and Diamond and Rajan (2001), where depositors design a fragile banking contract that will lead to a run if the banker tries to misappropriate funds. The moral hazard problem is different, of course; in our setting the incentives of the bank's investors as a group are distorted, rather than the incentives of a self-interested banker. But our model shares with these papers the idea that the threat of a run can partially mitigate distorted incentives.

6 Regulation

Our analysis so far allows the bank to choose any initial bail-in $h \geq 0$. The resulting outcome is inefficient: the initial bail-in is too small and bailouts are both too frequent and too large. The outcome can also involve a run on the bank in some cases. These results raise the question of whether regulation can improve outcomes. In this section, we show the regulator can raise welfare by restricting the values of h the bank is allowed to choose.

If the regulator could observe the bank's loss λ at the beginning of period 1, it would mandate the bank set the planner's initial bail-in, $h^*(\lambda)$. It is straightforward to show that the subsequent actions – the withdrawal decisions of investors, the bailout b, and the remaining bail-in $\hat{h}$ – would all match the planner's allocation as well. However, like the fiscal authority, the regulator observes bank-specific information only with a delay, after a fraction π of investors have withdrawn. The regulator wants the initial bail-in h to be a function of λ, which is the private information of the bank. It could give the bank discretion in setting h, but it knows the bank prefers to set the bail-in too low in some states. The choice of what restrictions to place on the bank can be formulated as a *delegation problem* following Holmström (1977, 1984) and others. We study this formulation below and characterize the optimal regulatory policy.

6.1 A delegation problem

As is standard in the delegation literature, our model has an uninformed principle (the regulator) who faces an informed agent (the bank). The bank is biased toward selecting a smaller bail-in, but only in states where it is bailed out.[18] We allow the regulator to choose a set of allowable bail-ins D, which can be any compact subset of the unit interval. The bank must then pick its initial initial bail-in h from this set. Restricting the bank's choice set is the regulator's only policy tool. In particular, the regulator cannot use state-contingent transfers to reward or punish the bank once the true state has been revealed. All transfers are made by the fiscal authority, as above, and are chosen to maximize *ex post* welfare.

After the regulator has chosen the delegation set D, the bank observes its realized loss λ and chooses an initial bail-in h from this set,

$$\max_{h \in D} W_B(h; \lambda). \tag{17}$$

Our analysis in the previous section corresponds to setting $D = [0, 1]$. Our first result in this section establishes that two basic properties of the solution to the bank's problem in the previous section carry over to an arbitrary closed subset D of the unit interval.

Proposition 8. *For any closed delegation set D, (i) a solution to the bank's maximization problem (17) exists for every $\lambda \in \Lambda$, and (ii) there exists $\lambda^e_D \in \Lambda$ such that the bank is bailed out in this solution if any only if $\lambda > \lambda^e_D$.*

The solution to the problem in equation (17) may not be unique; in some cases, the bank may be indifferent between two or more elements of D. In these cases, we assume the bank chooses the largest of the optimal values, which will minimize the associated bailout. Using this tie-breaking rule, the bank's optimal choice generates a function $h^e_D : \Lambda \to D$ that assigns an initial bail-in h^e to each possible loss λ. The regulator's optimization problem can then be expressed as

$$\max_{D} \mathcal{W}(\mathcal{D}) \equiv \int_0^{\bar{\lambda}} W_R\left(h^e_D(\lambda); \lambda\right) dF(\lambda). \tag{18}$$

If the bank were not bailed out in any state, the objectives of the regulator and the bank would be fully aligned. There would then be no benefit from restricting the bank's choice and *full delegation* ($D = [0, 1]$) would be optimal. At the other extreme, if the bank were bailed out in every state, *no delegation* would be optimal. The planner's bail-in would equal λ^* in

[18] In the standard delegation model, the agent's bias is exogenously determined by preferences. Here, in contrast, the set of states in which the bias appears is endogenously determined.

all states and the regulator could implement the efficient plan by giving the bank a single choice: $D = \{\lambda^*\}$. In our environment, where the bank is bailed out in some states and not in others, the regulator faces a tradeoff: more delegation is useful in states where the bank is not bailed out but costly in states where a bailout occurs. In the analysis that follows, we characterize the optimal choice of D and show that it represents *partial delegation*.

First, a technical issue: The regulator's problem in equation (18) will typically have many solutions. In particular, starting from one optimal set, removing elements the bank does not choose for any loss λ will lead to the same outcome. To simplify the analysis and facilitate interpretation of the results, we focus on the *largest* delegation set that solves the regulator's problem, that is, the set that imposes the fewest restrictions on the bank's choice. Let D^* denote the largest subset of $[0, 1]$ that solves the regulator's problem (18).

6.2 Optimal policy

We divide the analysis into cases based on the public sector's marginal costs of funds, as before. Our first result characterizes the optimal policy when the cost of funds is low.

Proposition 9. *If $\mu \leq \mu_1$, then $D^* = [h_1^*, 1]$ with $h_1^* > 0$.*

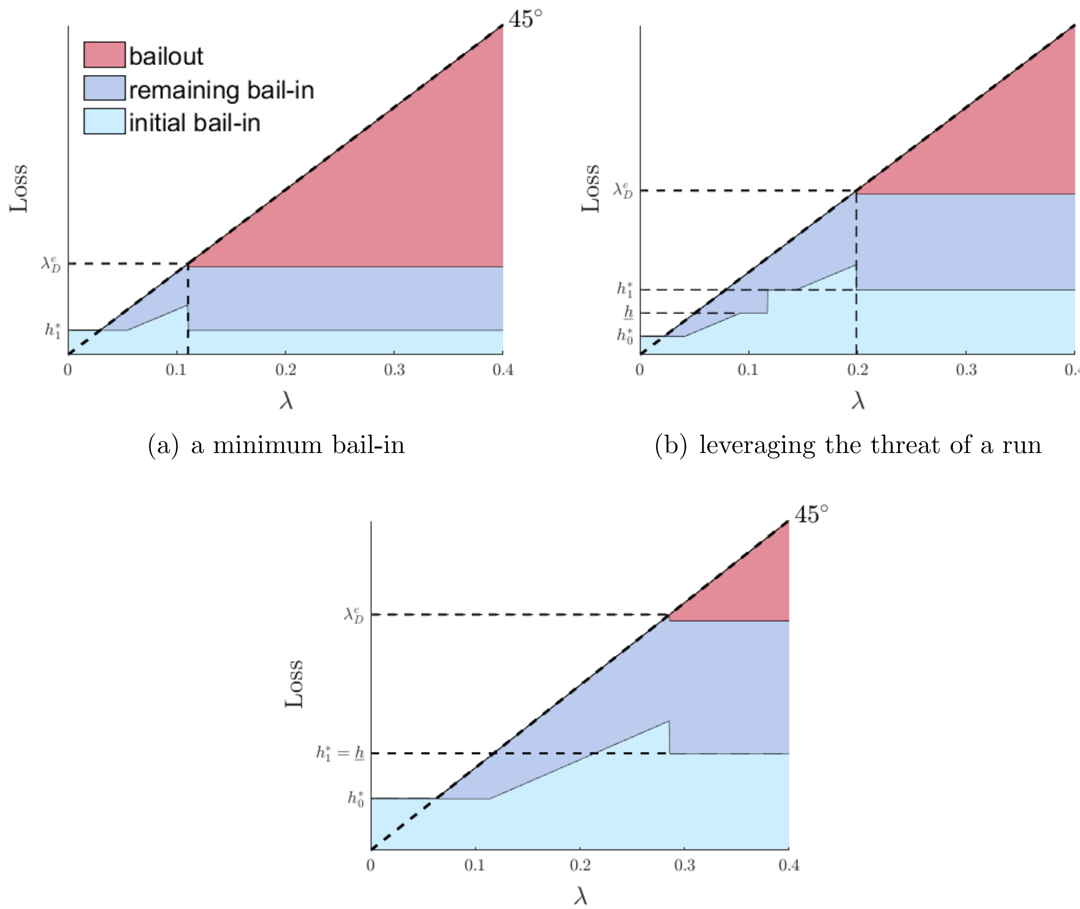
The optimal policy in this case takes a particularly simple form: a minimum bail-in $h_1^* > 0$. The bank is allowed to set h larger than h_1^*, and will do so in some states, but it must meet the minimum in all states. This type of result, where the optimal policy "caps" the agent's action against her bias, has been shown in the literature to arise in a range of settings.[19]

Panel (a) of Figure 5 presents the allocation of losses under the optimal policy for the same parameter values as in panel (a) of Figure 3, which satisfy $\mu < \mu_1$. Comparing the two panels highlights the policy's key effects. First, in states where the bank is bailed out, some of the loss now falls on the first π investors to withdraw (the light blue region), which makes the bailout (the red region) smaller. Second, the policy shifts the bailout cutoff λ_D^e to the right, meaning the bank is bailed out in fewer states. In this way, the policy decreases bailouts on both the intensive and extensive margins. The cost of the policy is the distortion it creates in states where the bank has a small or zero loss but must now impose a bail-in. The optimal value for the minimum bail-in h_1^* balances the benefits against this cost.

Proposition 9 shows that the optimal minimum bail-in is always strictly positive. Intuitively, the allocation of resources in states where the bank has little or no loss was efficient in the economy with no regulation. Increasing h_1 above zero thus initially has only has a second-order effect on investors' expected utility in these states. The welfare gain from

[19]See, for example, Amador and Bagwell (2013), Kartik et al. (2021), and the references therein.

imposing a bail-in in states where the bank is bailed out, in contrast, is first order. Notice that partial delegation is important for this result to obtain. If the regulator required the bank to set $h = h_1^*$ in all states, the cost of distorting the allocation in states where the bank is not bailed out would be first order and regulation might be undesirable. By allowing the bank to set a higher bail-in in some states, partial delegation decreases the cost of requiring a bail-in. Panel (a) of Figure 5 shows that the bank takes advantage of this option and chooses a bail-in larger than h_1^* in some states below the bailout cutoff.



(a) a minimum bail-in

(b) leveraging the threat of a run

(c) eliminating the run equilibrium

Figure 5: Optimal regulation

Our next result shows that when the cost of public funds is higher, the optimal policy may again be a minimum bail-in or may be more complex. A technical complication arises in this case. In some situations, the regulator would like banks to be able to choose bail-ins arbitrarily close to $\underline{h}$ as defined in equation (16), but not $\underline{h}$ itself. If the regulator chose such a set D, however, the bank's optimization problem in equation (17) will not have an exact solution in some states. To avoid this complication, we require the regulator to choose a closed set (in line with Proposition 8) and we study approximately-optimal policies that allow bail-ins within ε of $\underline{h}$, where ε is small.

Proposition 10. *If* $\mu > \mu_1$ *then* $D^* = [h_0, \underline{h} - \varepsilon] \cup [h_1, 1]$ *with* $0 \leq h_0^* \leq \underline{h} \leq h_1^* < 1$. *Moreover, at least one of* $h_0^* \geq 0$ *and* $h_1^* \geq \underline{h}$ *holds with strict inequality.*

The optimal delegation set when μ is large can be expressed as a union of two disjoint intervals. In some cases, h_0^* is large enough that the first interval is empty. The optimal policy is then a minimum bail-in h_1^*, as before, but with the minimum now bounded below by $\underline{h} > 0$, the smallest bail-in that prevents a run when the bank is in the bailout region. This form of policy will tend to be optimal when the probability of a small or zero loss is low, since the large minimum bail-in will substantially distort the allocation in those states, and when the probability of a loss in the bailout region is high.

In other cases, h_0^* is smaller and both intervals are non-empty. In these cases, the optimal policy leverages the disciplining effect of runs. Panel (b) of Figure 5 illustrates the idea, using the same parameter values as panel (b) of Figure 3. The lower bounds of the two intervals in D^* are selected so that, when the state is in the bailout region, the bank prefers choosing h_1^* and preventing a run over choosing h_0^* and suffering a run. In other words, the policy is calibrated so the threat of a run leads the bank to select h_1^* in this region even though smaller bail-ins are permitted. The bank chooses bail-ins from the lower interval only when it is strong enough that there is no bailout and investors have no incentive to run.

The regulator sets h_1^* strictly higher than $\underline{h}$ in this case, which implies D^* is not a connected set.[20] The higher value of h_1^* improves the allocation in the bailout region relative to Figure 3. However, increasing h_1 makes deviating to a lower bail-in more attractive to the bank. At h_1^*, the bank would choose $h = 0$ when it is in the bailout region if that choice were permitted. To prevent this deviation, the regulator increases h_0^* above zero, which has the side effect of distorting the allocation in states where the bank has a small or zero loss. The optimal policy balances these two concerns: the benefit that increasing h_1^* brings when the bank's loss is large and the cost that increasing h_0^* imposes when the loss is smaller.

In some cases, the optimal regulatory policy also enhances financial stability. In panel (c) of Figure 3, where there is no regulation, a run occurs when λ is in the bailout region. The bank could prevent this run by setting its bail-in to $\underline{h}$, but finds $h = 0$ more attractive. Panel (c) of Figure 5 depicts the optimal policy for the same parameter values. The regulator imposes a minimum bail-in $h_0^* > 0$, which makes choosing the lowest possible bail-in less attractive. In response, the bank sets its bail-in to $\underline{h} > h_0^*$ and prevents the run. The

[20] A non-connected optimal delegation set has been shown to arise in other settings as well. See for example Melumad and Shibano (1991) and Alonso and Matouschek (2008).

optimal policy sets h_0^* just high enough to induce this switch and sets $h_1^* = \underline{h}$. Focusing on the limit as $\varepsilon \to 0$, therefore, the policy corresponds to simple a minimum bail-in of h_0^*. The optimal minimum value h_0^* distorts the allocation in states where the bank has a small or zero loss, but this cost is more than offset by the gain from improving the allocation and preventing a run in states where the bank is bailed out.

This example also illustrates assessing the observed effects of a minimum bail-in policy can be tricky. In panel (c) of Figure 5, the minimum bail-in is binding only when the bank's loss is small. One might be tempted to conclude that the policy is ineffective: it is distorting the allocation when the loss is small, but is not binding in states where the loss is larger. This conclusion is incorrect, of course; absent the policy, the bank would choose $h = 0$ when the loss is larger and a run would occur, as shown in panel (c) of Figure 3.

Summary. The different forms of the optimal policy in the panels of Figure 5 should not obscure the common theme. Propositions 9 and 10 show that the optimal regulatory policy induces the bank to set a larger initial bail-in in those states where it is bailed out. This inducement can be direct, by imposing a minimum bail-in that is binding in those states, or indirect. The indirect approach induces the bank to choose a bail-in larger than the required minimum in these states by ensuring the threat of a run makes choosing any smaller allowable bail-in unattractive. In other words, optimal regulation leverages the threat of a run to discipline bank behavior. The optimal policy leads to larger initial bail-ins, which make bailouts smaller and less frequent, resulting in strictly higher welfare.

6.3 Discussion

System-wide bail-ins. As discussed above, our model would be unchanged if there were many banks, each of which received an idiosyncratic draw from the loss distribution F. In that case, the regulator's policy would be a *system-wide* bail-in based on a systemic trigger. In other words, the regulator would receive aggregate signal indicating some banks have experienced losses, but would not initially know how each bank is affected. The optimal policy is to promptly require all banks to choose a bail-in from the same delegation set D^*. Later, once policy makers are able to observe the loss in each bank, the bail-in $\hat{h}$ of the remaining investors and the bailout b would be tailored to bank-specific information.

Applications. Perhaps the most direct counterpart to our model in practice is a money market mutual fund, which has a single type of creditor (the shareholders) and allows these creditors to withdraw on a daily basis. Runs on certain types of money market funds in 2008 and again in 2020 have led to an ongoing policy debate over how to best reform or regulate these funds. Ennis et al. (2023) review the issues involved and identify the expectation of government support – bailouts – as a fundamental source of instability in this industry. Our model captures this concern and prescribes a course of action. Rather than relying on funds to voluntarily bail in their investors, as in the previous round of reforms, regulators should impose bail-in rules at the onset of periods of market stress. These rules should be designed to increase the bail-in chosen in states where the fund is weak while minimizing the distortion in states where it is sound. Our results in Propositions 9 and 10 show how to optimally balance these concerns.

We believe these lessons also apply to other types of financial intermediaries, which may have multiple types of creditors with different maturity and seniority of claims. The key distortion in our model is that the intermediary – and its creditors/investors – have an incentive to allow too many resources to flow out during the early stages of a crisis, before policy makers have actionable information. These payouts can take a variety of forms, including dividends and share repurchases in addition to repayment of maturing term debt and withdrawals. Regulating against this behavior is challenging precisely because the regulator does not yet know the extent of the problem. Nevertheless, our results indicate that prompt regulatory action is desirable. This action should restrict the choices available to intermediaries, while leaving them some discretion to tailor the bail-in to their situation.

Implementing the optimal policy. We have framed our analysis as a delegation problem, where the choice of initial bail-in h is made by the bank within the limits set by the regulator. There is an equivalent mechanism-design formulation in which the bank reports its private information and the regulator assigns an action based on this report. This interpretation may be more natural in some settings. In times of stress, where the bank may have experienced a loss, the regulator asks the bank to report its loss λ. The regulator then assigns the bail-in h that solves the problem in equation (17) based on the optimal delegation set D^*. The resulting allocation rule $h(\lambda)$ corresponds to the upper bound of the light-blue region in the panels of Figure 5. As the panels show, this rule is a non-monotone function of the bank's loss with one or more discontinuities Nevertheless, the rule is incentive compatible by design and the bank will report its loss truthfully.

Incentive to deposit. Requiring that banks bail-in their creditors in some states of nature may decrease the incentive for these investors to deposit with intermediaries in the first place. This effect does not appear in our model because investors save their entire (fixed) endowment. In a broader setting where agents make a consumption-savings choice, for example, a policy of mandatory bail-ins for banks may encourage agents to consume more and save less. In other words, regulatory bail-ins may result in a smaller banking system. In such cases, however, the smaller banking system would likely be socially *desirable*. When bailouts are larger than in the efficient plan, agents will have an incentive to save too much and the banking system (including shadow banks) will tend to be larger than in the planner's allocation. Reducing the incentive to deposit in such arrangements would thus be another dimension in which a mandatory bail-in policy could improve the allocation of resources.

7 Concluding remarks

Several policy reforms implemented in response to the financial crisis of 2008 aim to give financial intermediaries the ability to more easily impose losses on their investors and/or creditors without declaring bankruptcy or being placed in resolution. Examples include allowing money market mutual funds to restrict withdrawals and impose withdrawal fees, the introduction of swing pricing in the mutual fund industry more generally,[21] and the adoption of rules encouraging the issue of "bail-inable" bank debt. These reforms aim to allow intermediaries to better handle periods of financial stress without the need for bailouts or other forms of public support. While it remains to be seen how effective these reforms will be across a range of situations, the indications to date are not encouraging. At the onset of the Covid-19 crisis in the U.S. in March 2020, the Federal Reserve and U.S. Treasury moved quickly to "assist money market funds in meeting demands for redemptions" by creating a special facility to finance the purchase of assets from these funds.[22] The new tools designed for dealing with high redemption demand– restricting withdrawals or imposing withdrawal fees – were not used by any fund. This episode serves as a clear warning that financial-stability policies that rely on intermediaries choosing to quickly bail in their investors in periods of financial stress may be ineffective.

Our model captures the incentive problems that can undermine the effectiveness of these types of policies and points to a better approach. Banks and other intermediaries anticipate that, when the situation is bad enough, the public sector will respond with bailouts. It does not appear feasible for governments to commit to a strict no-bailout policy, and such a policy may not even be desirable; in our environment, it is optimal for the public sector to absorb some of the tail risk. The anticipation of being bailed out undermines the incentive for an intermediary to quickly bail in its investors and creditors, making initial bail-ins inefficiently small and bailouts inefficiently large. Moreover, these small initial bail-ins can be a source of fragility, triggering a run by investors that leads to an even larger bailout.

[21] See Chen et al. (2010) for evidence of strategic complementarities in the withdrawal decisions of investors in open-end mutual funds where the price is set daily according to the net asset value of the fund. Jin et al. (2022) study the ability of swing pricing to remove these complementarities and prevent runs.

[22] Detailed information on the Money Market Mutual Fund Liquidity Facility is available at `www.federalreserve.gov/monetarypolicy/mmlf.htm`.

Our results show how a regulator can improve outcomes by promptly imposing a bail-in policy with partial delegation. This policy may require the bank to set a minimum bail-in but allow the bank to choose within some bounds. The optimal policy is designed so that it increases the bail-in chosen by the bank in states where it will later be bailed out, even though the regulator does not initially observe the state. In some cases, the optimal policy takes advantage of the possibility of a run by investors to induce the bank to select an appropriate bail-in.

This type of policy can be implemented across a range of intermediation arrangements. In general terms, our results support restricting dividend payments and share repurchases by banks in the early stages of a crisis. Banks could also be required to issue debt that is either automatically written down or converted to equity based on a *systemic* trigger. Similarly, a minimum withdrawal fee could be imposed at all money market mutual funds based on measures of systemic stress that are available to policy makers in real time. One interesting area for future research is adapting our model to the specific institutional features of different intermediation arrangements and deriving the resulting prescriptions for bail-in policy for each arrangement.

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Supplemental Appendix: Proofs

Proposition 1. *The efficient plan satisfies $h^* = \hat{h}^*$ for all $\lambda \in \Lambda$.*

Proof. To begin, note that the resource constraint (4) will hold with equality for all λ at the solution to the planner's problem. The non-negativity restrictions then imply that the planner will set $h = \hat{h} = b = 0$ when $\lambda = 0$. When there is no loss, investors are neither bailed in nor bailed out.

For $\lambda > 0$, let θ denote the multiplier on the resource constraint (4). We can then write the first-order conditions for the optimal choice of h as[23]

$$u'\left((1-h)c_1^*\right) \geq \theta \quad \text{and} \quad h\left[u'\left((1-h)c_1^*\right) - \theta\right] = 0 \tag{19}$$

and for the optimal choice of $\hat{h}$ as

$$u'\left((1-\hat{h})c_2^*\right) \geq \frac{\theta}{R} \quad \text{and} \quad \hat{h}\left[u'\left((1-\hat{h})c_2^*\right) - \frac{\theta}{R}\right] = 0. \tag{20}$$

We will show that the solutions to these two sets of equations are necessarily the same, considering the cases of boundary and interior solutions separately.

First, suppose the solution has $h = 0$. Then equation (19) implies

$$u'\left(c_1^*\right) \geq \theta.$$

The reference allocation (c_1^*, c_2^*) is characterized by the standard optimality condition in the Diamond-Dybvig framework,

$$u'\left(c_1^*\right) = Ru'\left(c_2^*\right).$$

Combining these two equations yields

$$u'\left(c_2^*\right) \geq \frac{\theta}{R}$$

and, therefore, the unique $\hat{h}(\phi)$ satisfying the conditions in equation (20) is $\hat{h}(\phi) = 0$.

Next, suppose the solution has $h > 0$. Then equation (19) implies

$$u'\left((1-h)c_1^*\right) = \theta.$$

Given that the utility function u is of the constant-relative-risk-aversion form, the ratio of marginal utilities depends only on the ratio of consumption levels, that is, we have

$$\frac{u'\left(\alpha c_1^*\right)}{u'\left(\alpha c_2^*\right)} = \frac{u'\left(c_1^*\right)}{u'\left(c_2^*\right)} = R \tag{21}$$

[23]Note that the Inada conditions on the function u imply that the upper bounds on $h(\phi)$ and $\hat{h}(\phi)$ in equation (3) will never bind at the solution to the problem.

for any $\alpha > 0$. These last two equations imply

$$u'\left((1-h)c_2^*\right) = \frac{\theta}{R}$$

and, therefore, setting $\hat{h} = h$ is the unique solution to equation (20). Combining these two cases, we have shown that $\hat{h} = h$ holds for all λ, which establishes the result. □

Proposition 2. *The efficient plan* $\left(h^*, \hat{h}^*, b^*\right)$ *sets*

$$h^* = \hat{h}^* = \left\{ \begin{array}{c} \lambda \\ \lambda^* \end{array} \right\} \quad \textit{and} \quad b^* = \left\{ \begin{array}{c} 0 \\ \lambda - \lambda^* \end{array} \right\} \quad \textit{as} \quad \lambda \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} \lambda^*.$$

Proof. Using the result from Proposition 1 and the simplified resource constraint in equation (6), we can write the planner's problem as choosing the bail-in h to maximize

$$\pi u\big((1-h)c_1^*\big) + (1-\pi)u\big((1-h)c_2^*\big) - \mu[\lambda - h],$$

where the non-negativity constraints for bail-ins and bailouts can be written as

$$0 \leq h \leq \lambda. \tag{22}$$

The objective function is strictly concave in h and has slope

$$-\big[\pi u'\big((1-h)c_1^*\big)c_1^* + (1-\pi)u'\big((1-h)c_2^*\big)c_2^*\big] + \mu$$

or

$$-u'\big((1-h)c_1^*\big)\left[\pi c_1^* + (1-\pi)\frac{u'\big((1-h)c_2^*\big)}{u'\big((1-h)c_1^*\big)}c_2^*\right] + \mu.$$

Using equation (21) and the resource constraint for the reference allocation in equation (3), it is straightforward to show the term in square brackets reduces to 1. Using equation (7) to replace μ, we can then write the slope as

$$-u'\big((1-h)c_1^*\big) + u'\big((1-\lambda^*)c_1^*\big).$$

If $\lambda \leq \lambda^*$, this slope is non-negative when evaluated at the upper bound for h in equation (22) and, therefore, the solution is $h = \lambda$. If $\lambda > \lambda^*$, the constraints in equation (22) do not bind and the solution is $h^* = \lambda^*$. In both cases, the planner's optimal bailout is determined by setting $h = h^*$ in equation (6) and solving for b^*.

□

Proposition 3. *Given h and y, the bail-in of remaining investors $\hat{h}$ and bailout b satisfy:*
(i) if $\hat{h}_{NB}(h,y) \leq \lambda^*$, then $\hat{h}(h,y) = \hat{h}_{NB}(h,y)$ and $b(h,y) = 0$
(ii) if $\hat{h}_{NB}(h,y) > \lambda^*$, then $\hat{h}(h,y) = \lambda^*$ and and the bailout satisfies equation (13).

Proof. As a first step, we use equation (10) to write the bank's marginal value of resources after π withdrawals as

$$V_1\big(\psi(h,b),y\big) \equiv \left\{ \begin{array}{c} u'\big(R\psi(h,b)\big)R \\ \pi u'\big(\psi(h,b)c_1^*\big)c_1^* + (1-\pi)u'\big(\psi(h,b)c_2^*\big)c_2^* \end{array} \right\} \quad \text{as} \quad \left\{ \begin{array}{c} y=2 \\ y=1 \end{array} \right\}. \tag{23}$$

This expression together with the definition of ψ in equation (8) shows that V_1 is strictly decreasing in b for both values of y. In what follows, we use the expression to establish the two parts of the proposition in turn.

Part (i): First suppose $y=2$. Then $\hat{h}_{NB}(h,2) \leq \lambda^*$ implies

$$\big(1-\hat{h}_{NB}(h,2)\big)c_2^* \geq (1-\lambda^*)\, c_2^*.$$

Using equations (8) and (12), we can rewrite the left-hand side of the inequality in terms of the bank's remaining resources ψ,

$$R\psi(h,0) \geq (1-\lambda^*)\, c_2^*,$$

which implies

$$u'\big(R\psi(h,0)\big)R \leq u'\big((1-\lambda^*)\, c_2^*\big)R.$$

Using equation (21), we can rewrite the right-hand side of this inequality as

$$u'\big((1-\lambda^*)\, c_2^*\big)R = u'\big((1-\lambda^*)\, c_1^*\big) = \mu.$$

Combining these last two equations with equation (23) yields

$$V_1\big(\psi(h,0),2\big) \leq \mu,$$

which implies $b^*=0$ is the unique solution to the first-order condition in equation (11).

Now suppose $y=1$. In this case, we use $\hat{h}_{NB}(h,1) \leq \lambda^*$ to write both

$$\big(1-\hat{h}_{NB}(h,1)\big)c_1^* \geq (1-\lambda^*)\, c_1^* \quad \text{and} \quad \big(1-\hat{h}_{NB}(h,1)\big)c_2^* \geq (1-\lambda^*)\, c_2^*.$$

Using equations (8) and (12), we can write the left-hand side of these inequalities in terms of the bank's remaining resources ψ,

$$\psi(h,0)c_1^* \geq (1-\lambda^*)\, c_1^* \quad \text{and} \quad \psi(h,0)c_2^* \geq (1-\lambda^*)\, c_2^*.$$

The first of these two inequalities implies

$$u'\big(\psi(h,0)c_1^*\big)c_1^* \leq u'\big((1-\lambda^*)\, c_1^*\big)c_1^* = \mu c_1^*, \tag{24}$$

while the second implies

$$\begin{aligned} u'\big(\psi(h,0)c_2^*\big)c_2^* &\leq u'\big((1-\lambda^*)\, c_2^*\big)c_2^* \\ &= u'\big((1-\lambda^*)\, c_1^*\big)\frac{c_2^*}{R} = \mu\frac{c_2^*}{R} \end{aligned} \tag{25}$$

where the first equality on the second line uses equation (21). Combining equations (24) and (25) yields

$$\pi u'\big(\psi(h,0)c_1^*\big)c_1^* + (1-\pi)u'\big(\psi(h,0)c_2^*\big)c_2^* \leq \mu\big(\pi c_1^* + (1-\pi)\frac{c_2^*}{R}\big) = \mu,$$

where the last equality uses the resource constraint in equation (3). Combining this inequality with equation (23) yields

$$V_1\big(\psi(h,0),1\big) \leq \mu,$$

which implies $b^* = 0$ is the unique solution to the first-order condition in equation (11) when $y = 1$ as well. When $b^* = 0$, the remaining investors will be bailed in at rate $\hat{h}_{NB}$ as defined in equation (12).

Part (*ii*): When $\hat{h}_{NB}(h,1) > \lambda^*$, the steps above show that $V_1(\psi(h, b = 0), y) > \mu$ and, therefore, the solution to the fiscal authority's bailout choice problem is interior. In this case, the first-order condition in equation (11) holds with equality,

$$V_1\big(\psi(h,b),y\big) = \mu = u'\big((1-\lambda^*)\, c_1^*\big).$$

If $y = 2$, this equation can be written as

$$u'\big(R\psi(h,b)\big)R = u'\big((1-\lambda^*)\, c_1^*\big) = u'\big((1-\lambda^*)\, c_2^*\big)R,$$

where the last equality uses equation (21). Using the monotonicity of u', we have

$$R\psi(h,b) = (1-\lambda^*)\, c_2^*.$$

Using the definition of ψ in equation (8), we can rewrite this equation as

$$R\frac{1-\lambda-\pi(1-h)c_1^* + b}{1-\pi} = (1-\lambda^*)\, c_2^*.$$

Solving for b yields

$$\begin{aligned} b &= (1-\pi)(1-\lambda^*)\frac{c_2^*}{R} - (1-\pi)\left(\frac{1-\lambda-\pi(1-h)c_1^*}{1-\pi}\right) \\ &= (1-\pi)\frac{c_2^*}{R}\left(1-\lambda^* - \frac{R}{c_2^*}\,\frac{1-\lambda-\pi(1-h)c_1^*}{1-\pi}\right). \end{aligned}$$

Finally, using equations (3) and (12), we can rewrite this equation as

$$b = (1-\pi c_1^*)\left(\hat{h}_{NB}(h,2) - \lambda^*\right),$$

as desired.

The steps for $y = 1$ are similar. Equation 7 can be written as

$$\pi u'\big(\psi(h,b)c_1^*\big)c_1^* + (1-\pi)u'\big(\psi(h,b)c_2^*\big)c_2^* = u'\big((1-\lambda^*)\, c_1^*\big).$$

Using equation (21), we can write this equation as

$$u'\left(\psi(h,b)c_1^*\right)\left(\pi c_1^* + (1-\pi)\frac{c_2^*}{R}\right) = u'\left((1-\lambda^*)\,c_1^*\right).$$

Using equation (3) and the monotonicity of u', this equation implies

$$\psi(h,b) = 1-\lambda^*,$$

or, replacing ψ using equation (8),

$$\frac{1-\lambda-\pi(1-h)c_1^*+b}{1-\pi} = 1-\lambda^*.$$

Solving for b yields

$$\begin{aligned} b &= (1-\pi)\left(1-\lambda^* - \frac{1-\lambda-\pi(1-h)c_1^*}{1-\pi}\right) \\ &= (1-\pi)\left(\hat{h}_{NB}(h,1)-\lambda^*\right), \end{aligned}$$

as desired.

□

Proposition 4. *The function $y(h)$ in equation (14) is weakly increasing. The composite function $b\,(h, y(h))$ defined by equations (13) and (14) is decreasing in h and is strictly decreasing whenever $b(h, y(h)) > 0$.*

Proof. For the first part of the proposition, equation (12) shows that $\hat{h}_{NB}(h,2)$ is strictly decreasing in h. The right-hand side of the inequality in equation (14) is, therefore, weakly increasing in h, while the left-hand side is strictly decreasing. Moreover, $\lambda^* < 1$ implies $y(h) = 2$ will always hold for h sufficiently close to 1. If follows that either (i) $y(h) = 2$ for all $h \in [0,1]$ or (ii) $y(h) = 1$ for $h < x$ and $y(h) = 2$ for $h \geq x$ for some $x \in (0,1)$; in both cases, $y(h)$ is weakly increasing. Intuitively, a larger bail-in always decreases the incentive for patient investors to run.

For the second part of the proposition, we first show $b(h,y)$ is decreasing in h holding y fixed. For either value of y, equation (12) shows $\hat{h}_{NB}(h,y)$ is strictly decreasing in h. Equation (13) then shows that for any $h' > h$, we have $b(h',y) \leq b(h,y)$ for any y, with strict inequality if $b(h,y) > 0$. Intuitively, if the bank paid less to the investors who have already withdrawn, it receives a smaller bailout.

We next show $b(h,y)$ is decreasing in y. Using $c_2^* < R$ in equation (12) shows that $\hat{h}_{NB}(h,y)$ decreases as we move from $y = 1$ to $y = 2$ for any h. Using this fact in equation (13), together with $c_1^* > 1$, implies we have $b(h,2) \leq b(h,1)$ for any h. Intuitively, the bank receives a smaller bailout if there is no run.

Combining these two results with the first part of the proposition shows that for any $h' > h$, we have

$$b\,(h, y(h)) \geq b\,(h', y(h)) \geq b\,(h', y(h')),$$

where the first inequality is strict if $b\,(h, y(h)) > 0$. Intuitively, a higher bail-in h leaves the bank with more resources and may, in addition, prevent a run. Both of these effects decrease the bailout payment it receives.

□

Proposition 5. *There exists $\lambda_1^e < \lambda^*$ such that, when $\mu \leq \mu_1$, the bank is bailed out if and only if $\lambda > \lambda_1^e$. In this region, the bank sets $h^e = 0$, patient investors do not run ($y^e = 2$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + \lambda^* \pi c_1^* \quad > \; b^*.$$

Proof. In states where the bank is bailed out, it will set its initial bail-in h either to zero or to the lowest value that prevents a run, $\underline{h}$. The cutoff μ_1 is defined so that $\mu \leq \mu_1$ implies $\underline{h} = 0$; it follows immediately that the bank will set $h^e = 0$ in these states and that patient investors do not run ($y^e = 2$). ix or

$$b^e = (1 - \pi c_1^*)(1 - \lambda^*) - (1 - \pi c_1^*)\frac{R}{(1-\pi)c_2^*}(1 - \lambda - \pi c_1^*).$$

Using the resource constraint in equation (3) and regrouping terms yields

$$b^e = (1 - \lambda^*) - (1 - \lambda^*)\pi c_1^* - (1 - \lambda - \pi c_1^*)$$

or

$$b^e = \lambda - \lambda^* + \lambda^* \pi c_1^*,$$

as stated in the proposition. The planner's bailout b^* is shown in Proposition 2 to equal $\lambda - \lambda^*$. Since λ^*, π, and c_1^* are all strictly positive, the decentralized bailout is strictly larger than b^*.

What remains is to be shown that (i) there exists a cutoff λ_1^e such that the bank is bailed out if and only if $\lambda > \lambda_1^e$ and (ii) this cutoff is below the efficient level λ^*. In states where the bank is not bailed out, it will set $h = \hat{h} = \lambda$. The bank will choose $h = 0$, and hence be bailed out, if and only if doing so yields higher expected utility, that is,[24]

$$\pi \underbrace{u\,(c_1^*)}_{h=0} + (1-\pi)\underbrace{u\,((1-\lambda^*)c_2^*)}_{\text{bailed out}} > \pi \underbrace{u\,((1-\lambda)c_1^*)}_{h=\lambda} + (1-\pi)\underbrace{u\,((1-\lambda)c_2^*)}_{\text{not bailed out}}$$

Using the form of the utility function in equation (1), we can factor out the $(1 - \lambda)$ term on the right-hand side,

$$\pi \; u\,(c_1^*) + (1-\pi)u\big((1-\lambda^*)c_2^*\big) > (1-\lambda)^{1-\gamma}\big(\pi u\,(c_1^*) + (1-\pi)u\,(c_2^*)\big)$$

[24]The inequality is strict because we assume the bank chooses the larger bail-in if it is exactly indifferent.

and solve for

$$\lambda > 1 - \left(\frac{\pi \; u\left(c_1^*\right) + (1-\pi) u\left((1-\lambda^*) c_2^*\right)}{\pi u\left(c_1^*\right) + (1-\pi) u\left(c_2^*\right)} \right)^{\frac{1}{1-\gamma}} \equiv \lambda_1^e.$$

To compare λ_1^e with λ^*, we use the explicit solution to the planner's problem,

$$c_1^* = \frac{1}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} \quad \text{and} \quad c_2^* = \frac{R^{\frac{1}{\gamma}}}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} \tag{26}$$

to obtain

$$\lambda_1^e = 1 - \left(\frac{\pi + (1-\pi)(1-\lambda^*)^{1-\gamma} R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} \right)^{\frac{1}{1-\gamma}}$$

Our assumption in equation (7) implies $\lambda^* > 0$ and, therefore,

$$\begin{aligned} \lambda_1^e &< 1 - \left(\frac{\pi(1-\lambda^*)^{1-\gamma} + (1-\pi)(1-\lambda^*)^{1-\gamma} R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} \right)^{\frac{1}{1-\gamma}} \\ &= 1 - \left((1-\lambda^*)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \\ &= \lambda^*, \end{aligned}$$

as desired.

□

Proposition 6. *There exist $\mu_2 > \mu_1$ and $\lambda_2^e < \lambda^*$ such that, when $\mu_1 < \mu < \mu_2$, the bank is bailed out if and only if $\lambda > \lambda_2^e$. In this case, the bank sets $h^e = \underline{h} > 0$, patient investors do not run ($y^e = 2$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + \left(\lambda^* - \underline{h}\right) \pi c_1^* \quad > \; b^*.$$

Proof. When $\mu > \mu_1$, $\underline{h}$ is strictly positive. In states where it is bailed out, the bank must choose between setting $h = \underline{h}$ to prevent a run and setting $h = 0$ and provoking a run. If will choose $h = \underline{h}$ if

$$\begin{aligned} &\pi \underbrace{u\Big((1-\underline{h}) c_1^*\Big)}_{h=\underline{h}} + (1-\pi) \underbrace{u\Big((1-\lambda^*) c_2^*\Big)}_{\text{all patient}} \\ &\qquad \geq \pi \underbrace{u\left(c_1^*\right)}_{h=0} + (1-\pi) \underbrace{\Big(\pi u\Big((1-\lambda^*) c_1^*\Big) + (1-\pi) u\Big((1-\lambda^*) c_2^*\Big) \Big)}_{\text{mix of impatient and patient}} \end{aligned} \tag{27}$$

The definition of $\underline{h}$ in equation (16) implies

$$(1-\underline{h})c_1^* = (1-\lambda^*)c_2^*, \tag{28}$$

so we can write the previous inequality as

$$u\Big((1-\lambda^*)c_2^*\Big) \geq \pi u\left(c_1^*\right) + (1-\pi)\Big(\pi u\Big((1-\lambda^*)c_1^*\Big) + (1-\pi)u\Big((1-\lambda^*)c_2^*\Big)\Big)$$

Using the form of the utility function in equation (1), we have

$$(1-\lambda^*)^{1-\gamma}u(c_2^*) \geq \pi u\left(c_1^*\right) + (1-\pi)(1-\lambda^*)^{1-\gamma}\Big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\Big)$$

or, bearing in mind that $\gamma > 1$ and $u(\cdot) < 0$,

$$(1-\lambda^*)^{1-\gamma} \leq \frac{u(c_1^*)}{(2-\pi)u(c_2^*) - (1-\pi)u(c_1^*)}.$$

Again using $\gamma > 1$, this expression can be written as

$$1-\lambda^* \geq \left(\frac{u(c_1^*)}{(2-\pi)u(c_2^*) - (1-\pi)u(c_1^*)}\right)^{\frac{1}{1-\gamma}}$$

Next, we use equation (7) to replace λ^* on the left-hand side and equation (26) to replace c_1^* and c_2^* on the right-hand side, we have

$$\frac{1}{\mu^{\frac{1}{\gamma}}c_1^*} \geq \left(\frac{1}{(2-\pi)R^{\frac{1-\gamma}{\gamma}} - (1-\pi)}\right)^{\frac{1}{1-\gamma}}$$

or

$$\mu^{\frac{1}{\gamma}}c_1^* \leq \left((2-\pi)R^{\frac{1-\gamma}{\gamma}} - (1-\pi)\right)^{\frac{1}{1-\gamma}}$$

or

$$\mu \leq R\left(c_1^*\right)^{-\gamma} \underbrace{\left((2-\pi) - (1-\pi)R^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{1-\gamma}}}_{>1} \equiv \mu_2.$$

Given $\mu_1 = (c_1^*)^{-\gamma}$, the expression above shows $\mu_2 > \mu_1$ holds.

The argument above establishes that when $\mu \in (\mu_1, \mu_2]$, the bank will set $h = \underline{h} > 0$ and patient investors will not run ($y^e = 2$) in states where the is bailed out. Substituting these values into the expression for $b(h, y)$ in equation (13) yields

$$b^e = (1-\pi c_1^*)\left(1 - \frac{R}{c_2^*}\frac{1-\lambda-\pi(1-\underline{h})c_1^*}{1-\pi} - \lambda^*\right).$$

We can rewrite this expression as

$$b^e = (1 - \pi c_1^*)(1 - \lambda^*) - (1 - \pi c_1^*)\frac{R}{(1-\pi)c_2^*}(1 - \lambda - \pi(1 - \underline{h})c_1^*),$$

or, using the resource constraint in equation (3) and regrouping terms,

$$b^e = (1 - \lambda^*) - (1 - \lambda^*)\pi c_1^* - (1 - \lambda) + \pi(1 - \underline{h})c_1^*),$$

which simplifies to

$$b^e = \lambda - \lambda^* + (\lambda^* - \underline{h})\pi c_1^*.$$

Using $b^* = \lambda - \lambda^*$ and $\underline{h} < \lambda^*$, we have $b^e > b^*$.

What remains is to show that (i) there exists a cutoff λ_2^e such that the bank is bailed out if and only if $\lambda > \lambda_2^e$ and (ii) this cutoff is below the efficient level λ^*. The bank will choose $h = \underline{h}$, and hence be bailed out, rather than setting $h = \hat{h} = \lambda$ if and only if

$$\pi \underbrace{u\left((1-\underline{h})c_1^*\right)}_{h=\underline{h}} + (1-\pi)\underbrace{u\left((1-\lambda^*)c_2^*\right)}_{\text{bailed out}} > \pi \underbrace{u\left((1-\lambda)c_1^*\right)}_{h=\lambda} + (1-\pi)\underbrace{u\left((1-\lambda)c_2^*\right)}_{\text{not bailed out}}$$

Using equation (28) and the form of the utility function in equation (1), we can write this inequality as

$$(1-\lambda^*)^{1-\gamma} u\left(c_2^*\right) > (1-\lambda)^{1-\gamma}\left(\pi u\left(c_1^*\right) + (1-\pi)u\left(c_2^*\right)\right)$$

or, bearing in mind that $\gamma > 1$ and $u(\cdot) < 0$,

$$1 - \lambda < (1-\lambda^*)\left(\pi\frac{u(c_1^*)}{u(c_2^*)} + (1-\pi)\right)^{\frac{1}{\gamma-1}}$$

or

$$\lambda > 1 - (1-\lambda^*)\underbrace{\left(\pi\frac{u(c_1^*)}{u(c_2^*)} + (1-\pi)\right)^{\frac{1}{\gamma-1}}}_{<1} \equiv \lambda_2^e.$$

Note that $\lambda_2^e < 1 - (1 - \lambda^*) = \lambda^*$ is immediate from the expression above.

□

Proposition 7. *There exists $\lambda_3^e < \lambda^*$ such that, when $\mu > \mu_2$, the bank is bailed out if and only if $\lambda > \lambda_3^e$. In this case, the bank sets $h^e = 0$, patient investors run ($y^e = 1$), and the equilibrium bailout payment is*

$$b^e = \lambda - \lambda^* + \lambda^*\pi c_1^* + (1 - \lambda^*)\pi(c_1^* - 1).$$

Proof. When $\mu > \mu_2$, the proof of Proposition 6 shows that, in states where the bank is bailed out, the inequality in equation (27) is reversed, so the bank will set $h^e = 0$ and patient investors will run ($y^e = 2$). Substituting these values into the expression for $b(h, y)$ in equation (13) yields

$$
\begin{aligned}
b^e &= (1-\pi)\left(1 - \frac{1-\lambda-\pi c_1^*}{1-\pi} - \lambda^*\right) \\
&= \lambda - \lambda^* + \pi\big(c_1^* - (1-\lambda^*)\big) \\
&= \lambda - \lambda^* + \lambda^*\pi c_1^* + \underbrace{(1-\lambda^*)\pi(c_1^*-1)}_{\text{extra due to run}}.
\end{aligned}
$$

This expression makes clear that b^e is strictly greater than $b^* = \lambda - \lambda^*$.

What remains is to be shown that (i) there exists a cutoff λ_3^e such that the bank is bailed out if and only if $\lambda > \lambda_1^e$ and (ii) this cutoff is below the efficient level λ^*. The bank will set $h = 0$ and be bailed out rather than choosing $h = \lambda$ if and only if

$$
\pi \underbrace{u(c_1^*)}_{h=0} + (1-\pi)\underbrace{\Big(\pi u\big((1-\lambda^*)c_1^*\big) + (1-\pi)u\big((1-\lambda^*)c_2^*\big)\Big)}_{\text{mix of impatient and patient; bailed out}}
$$

$$
> \pi \underbrace{u\left((1-\lambda)c_1^*\right)}_{h=\lambda} \quad + \quad (1-\pi) \underbrace{u\left((1-\lambda)c_2^*\right)}_{\text{all patient; no bailout}}
$$

Using the form of the utility function in equation (1), we can write this inequality as

$$
\pi u(c_1^*) + (1-\lambda^*)^{1-\gamma}(1-\pi)\big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\big) > (1-\lambda)^{1-\gamma}\big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\big)
$$

or

$$
(1-\lambda)^{1-\gamma} > \frac{\pi u(c_1^*) + (1-\lambda^*)^{1-\gamma}(1-\pi)\big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\big)}{\pi u(c_1^*) + (1-\pi)u(c_2^*)}
$$

or, since $\gamma > 1$,

$$
(1-\lambda) < \left(\frac{\pi u(c_1^*) + (1-\pi)u(c_2^*)}{\pi u(c_1^*) + (1-\lambda^*)^{1-\gamma}(1-\pi)\big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\big)}\right)^{\frac{1}{\gamma-1}}
$$

or

$$
\lambda > 1 - \left(\frac{\pi u(c_1^*) + (1-\pi)u(c_2^*)}{\pi u(c_1^*) + (1-\lambda^*)^{1-\gamma}(1-\pi)\big(\pi u(c_1^*) + (1-\pi)u(c_2^*)\big)}\right)^{\frac{1}{\gamma-1}} \equiv \lambda_3^e.
$$

To compare λ_3^e with the efficient bailout cutoff λ^*, we use the fact that $\mu > \mu_2$ implies the inequality in equation (27) is reversed, which implies

$$\lambda_3^e < 1 - \left(\frac{\pi u(c_1^*) + (1-\pi)u(c_2^*)}{\pi u\big((1-\underline{h})c_1^*\big) + (1-\pi)u\big((1-\lambda^*)c_2^*\big)} \right)^{\frac{1}{\gamma-1}}.$$

Using equation (28) to replace $\underline{h}$, we have

$$\lambda_3^e < 1 - \left(\frac{\pi u(c_1^*) + (1-\pi)u(c_2^*)}{(1-\lambda^*)^{1-\gamma} u(c_2^*)} \right)^{\frac{1}{\gamma-1}}.$$

$$= 1 - \left(\pi \frac{u(c_1^*)}{u(c_2^*) + (1-\pi)} \right)^{\frac{1}{\gamma-1}} (1-\lambda^*)$$

$$< 1 - (1-\lambda^*) \quad = \lambda^*.$$

We have, therefore, established that the decentralized bailout cutoff λ_3^e is below the efficient cutoff λ^*, as desired.

□

Proposition 8. *For any closed delegation set D, (i) a solution to the bank's maximization problem (17) exists for every $\lambda \in \Lambda$, and (ii) there exists $\lambda_D^e \in \Lambda$ such that the bank is bailed out in this solution if any only if $\lambda > \lambda_D^e$.*

Proof. Part (i): For any fixed $\lambda \in \Lambda$, the function $W_B(h;\lambda)$ defined in equation (15) is continuous in h except at points where $y(h)$ changes value. Proposition 4 shows that $y(h)$ changes value at most once as h increases from 0 to 1: if $y(0) = 1$, the value changes to $y(h) = 2$ when h reaches the point where withdrawing early is no longer a strictly dominant strategy. As a result, W_B is an upper semi-continuous function of h on the unit interval and, therefore, attains a maximum on any compact subset D.

Part (ii): Suppose the bank is not bailed out under its optimal choice $h_D^e(\lambda)$ for some λ. We will show that the bank is also not bailed out under its optimal choice for any $\lambda' < \lambda$. It then follows that the set of ϕ for which the bank is not bailed out is an interval of the form $[0, \lambda_D^e]$ for some $\lambda_D^e \in \Lambda$.

Equations (12) and (14) shows that when λ decreases, the set of h that lead to a bailout becomes weakly smaller. If there is no choice $h \in D$ that leads to a bailout for realization λ, therefore, the same is true for any $\lambda' < \lambda$ and the result is established. If there are some $h \in D$ that lead to a bailout in state λ, let $\hat{h}(\lambda)$ denote the best such choice. Since h_D^e is an optimal choice, we clearly have

$$W_B(h_D^e(\lambda);\lambda) \geq W_B(\hat{h}(\lambda);\lambda).$$

Now consider any $\lambda' < \lambda$. It is straightforward to show that $W_B(h;\lambda)$ is non-increasing in λ (holding h fixed) so we have

$$W_B(h_D^e(\lambda');\lambda') \geq W_B(h_D^e(\lambda);\lambda') \geq W_B(h_D^e(\lambda);\lambda).$$

What remains to be shown is that $h^e_D(\lambda')$ does not lead to a bailout. Let $\hat{h}(\lambda')$ denote the best choice that does lead to a bailout.[25] Because the set of h that lead to a bailout is increasing in λ, $\hat{h}(\lambda')$ would have also led to a bailout under the original realization. Moreover, when the bank is bailed out, its payoff is independent of λ, so we have

$$W_B(\hat{h}(\lambda');\lambda') = W_B(\hat{h}(\lambda');\lambda) \leq W_B(\hat{h}(\lambda);\lambda).$$

Combining the above inequalities shows $W_B(h^e_D(\lambda');\lambda') \geq W_B(\hat{h}(\lambda');\lambda')$, meaning the bank is not bailed out under its optimal choice for λ' and we have established the result.

□

Proposition 9. *If $\mu \leq \mu_1$, then $D^* = [h^*_1, 1]$ with $h^*_1 > 0$.*

Proof. The proof is divided into two steps. We first show the optimal delegation set must be an interval of the form $[h_1, 1]$ for some $h_1 \geq 0$. We then show this lower bound is strictly positive.

Step (i): Show $D^* = [h_1, 1]$ for some $h_1 \geq 0$.

Given any compact delegation set D, define a new set $\hat{D} \equiv [h_1, 1]$ where h_1 is the smallest element of D. To establish this step, we show that $\hat{D}$ weakly dominates D, that is, $\mathcal{W}(\hat{D}) \geq \mathcal{W}(D)$. It then follows that the largest optimal delegation set D^* must also have the form.

Because $\hat{D}$ contains D, the bank's optimized payoff must be at least as high,

$$W_B\left(h^e_{\hat{D}}(\lambda);\lambda\right) \geq W_B\left(h^e_D(\lambda);\lambda\right) \quad \text{for all } \lambda \in \Lambda. \tag{29}$$

The regulator's payoff equals W_B plus the cost of any bailout payment. To establish that the regulator's payoff is also at least as high, we will show that the bailout associated with $h^e_{\hat{D}}(\lambda)$ is no larger than the bailout associated with $h^e_D(\lambda)$ for all λ.

Consider first any λ such that $h^e_D(\lambda) = h_1$. In these cases, the bank's choice of h cannot decrease when we move to policy $\hat{D}$. Using the second part of Proposition 4, therefore, the bailout payment cannot increase, that is

$$b\left(h^e_{\hat{D}}(\lambda), 2\right) \leq b\left(h^e_D(\lambda), 2\right) \text{ for any } \lambda \text{ such that } h^e_D(\lambda) = h_1. \tag{30}$$

Next consider any λ such that $h^e_D(\lambda) > h_1$. In these states, the bank is not bailed out under policy D. While the bank's optimal choice of bail-in h may decrease when we move to policy $\hat{D}$, the bailout must remain zero. To see why, suppose this were not true, that is, suppose the bank were bailed out following $h^e_{\hat{D}}(\lambda)$. Because $\mu \leq \mu_1$, we know $\underline{h}$ as defined in equation (16) is zero and no choice of bail-in will lead to a run. If the bank is being bailed out, therefore, it must be choosing the smallest element of $\hat{D}$, that is, $h^e_{\hat{D}}(\lambda) = h_1$. But h_1 was a feasible choice under policy D as well, which contradicts the fact that $h^e_D(\lambda) > h_1$ was

[25] If no such choice exists for realization λ', the bank is clearly not bailed out and the result is established.

chosen.[26] We thus have

$$b\left(h^e_{\hat{D}}(\lambda), 2\right) = b\left(h^e_D(\lambda), 2\right) = 0 \text{ for any } \lambda \text{ such that } h^e_D(\lambda) > h_1. \tag{31}$$

Combining equations (29) – (31) yields

$$W_R\left(h^e_{\hat{D}}(\lambda); \lambda\right) \geq W_R\left(h^e_D(\lambda); \lambda\right) \text{ for all } \lambda \in \Lambda. \tag{32}$$

Using the definition of $\mathcal{W}$ in equation (18), we then have $\mathcal{W}(\hat{D}) \geq \mathcal{W}(D)$, as desired.

Step (ii): Show $h^*_1 > 0$.

Because the optimal delegation set has the form $[h_{min}, 1]$, we can write the regulator's expected payoff as

$$\int_0^{h_1} W_R\left(h_1; \lambda\right) dF(\lambda) + \int_{h_1}^{\lambda^e_D} W_R\left(\lambda; \lambda\right) dF(\lambda) + \int_{\lambda^e_D}^{\bar{\lambda}} W_R\left(h_1; \lambda\right) dF(\lambda).$$

If the bank has a zero or small loss, it chooses the smallest allowable bail-in, h_1. When the loss is between h_1 and the bailout cutoff λ^e_D, the bank sets $h = \lambda$ and is not bailed out. When the loss is larger than λ^e_D, the bank chooses the smallest allowable bail-in and is bailed out. The bailout cutoff also depends on h_1 and can be shown in this case to be

$$\lambda^e_D\left(h_1\right) = 1 - \left(\frac{\pi u\left((1-h_1)c^*_1\right) + (1-\pi)u\left(\phi^* c^*_2\right)}{\pi u\left(c^*_1\right) + (1-\pi)u\left(c^*_2\right)}\right)^{\frac{1}{1-\gamma}}. \tag{33}$$

It is straightforward to show this cutoff is increasing in h_1. When the minimum bail-in is larger, being bailed out is less attractive to the bank and the set of states in which a bailout occurs shrinks.

Because the distribution F may put positive probability on $\lambda = 0$, it is useful to rewrite the regulator's payoff using the density function f. Letting $z \geq 0$ denote the probability of $\lambda = 0$, we have

$$z\, W_R\left(h_1; 0\right) + \int_0^{h_1} W_R\left(h_1; \lambda\right) f(\lambda) d\lambda + \int_{h_1}^{\lambda^e_D} W_R\left(\lambda; \lambda\right) f(\lambda) d\lambda + \int_{\lambda^e_D}^{\bar{\lambda}} W_R\left(h_1; \lambda\right) f(\lambda) d\lambda. \tag{34}$$

Investors never have an incentive to run when $\mu < \mu_1$, meaning $y(h) = 2$ holds for all h. In this case, the function $W_R(h; \lambda)$ is continuous in h and is differentiable for all λ except the bailout cutoff λ^e_D. We can, therefore, write the slope of the regulator's expected payoff in

[26] Recall that, if the bank were indifferent between $h = h_1$ and $h = h^e_D(\lambda) > h_1$, it would have chosen the larger bail-in under our tie-breaking rule.

equation (34) with respect to h_1 as

$$z\,\frac{dW_R}{dh}\left(h_1;0\right) \;+\; \int_0^{h_1}\frac{dW_R}{dh}\left(h_1;\lambda\right)f(\lambda)d\lambda \tag{35}$$
$$+\Big[W_R\left(\lambda_D^e;\lambda_d^e\right)-W_R\left(h_1,\lambda_D^e\right)\Big]f(\lambda_D^e)\frac{d\lambda_D^e}{dh_1} \;+\; \int_{\lambda_D^e}^{\bar{\lambda}}\frac{dW_R}{dh}\left(h_1;\lambda\right)f(\lambda)d\lambda.$$

The first two terms in this expression capture the cost of raising h_1: it increases the distortion in states where the bank has no loss or only a small loss. The last two terms capture the benefit of increasing h_1: it shrinks the set of states where the bank is bailed out and increases the bail-in the bank must use in those states. To evaluate this slope at $h_1 = 0$, we write out the first term as

$$\frac{dW_R}{dh}\left(h_1;0\right)=\pi c_1^*\left(-u'\left((1-h_1)c_1^*\right)+Ru'\left(\frac{R}{1-\pi}(1-\pi(1-h_1)c_1^*)\right)\right).$$

Evaluating this term at $h_1 = 0$ yields

$$\frac{dW_R}{dh}\left(0;0\right)=\pi c_1^*\Big(-u'\left(c_1^*\right)+Ru'\left(c_2^*\right)\Big)=0. \tag{36}$$

In other words, as h_1 increases from zero, the cost of the distortion when the bank has no loss is second-order because the bank was at an unconstrained optimum. The second term in equation (35) also vanishes when $h_1 = 0$. The third and fourth terms, in contrast, remain strictly positive. It follows that the regulator's objective function is strictly increasing at $h_1 = 0$ and, therefore, the optimal choice h_1^* is strictly positive.

□

Proposition 10. *If $\mu > \mu_1$ then $D^* = [h_0, \underline{h} - \varepsilon] \cup [h_1, 1]$ with $0 \le h_0^* \le \underline{h} \le h_1^* < 1$. Moreover, at least one of $h_0^* > 0$ and $h_1^* > \underline{h}$ holds with strict inequality.*

Proof. We follow a similar approach to that in the proof of Proposition 9. Given any delegation set D, we first define another set $\hat{D}$ that contains D and is the union of two intervals, as in the statement of the proposition. We show that $\hat{D}$ generates a payoff at least as high as D and, therefore, the optimal delegation set must have this form. We then establish that at least one of the inequalities in the proposition is strict.

Step 1: Define the new set $\hat{D}$.

Given any D, let h_1 denote its smallest element satisfying $h_1 \ge \underline{h}$. In other words, h_1 is the smallest bail-in the bank can choose when it is in the bailout region without causing a run. Because $\mu > \mu_1$, we have $\underline{h} > 0$ and, hence, h_1 is strictly positive as well. Let h_0 denote the smallest overall element of D. Define

$$\hat{D} = [h_0, \underline{h}) \cup [h_1, 1]\,. \tag{37}$$

Note that $\hat{D}$ contains the original delegation set D by construction. It consists of two disjoint intervals. All h in the lower interval would cause a run if chosen when the bank is in the bailout region, while all h in the upper interval would prevent a run. In states where the bank is bailed out, it will choose the smallest element of one of these two intervals, that is, either h_0 or h_1.

Step 2: Show $\mathcal{W}(\hat{D}) \geq \mathcal{W}(D)$.

Consider first an intermediate delegation set,

$$D' = D \cup [h_1, 1].$$

That is, suppose we add to D only those bail-in choices that lie above h_1. The argument that moving from D to D' cannot decrease the regulator's payoff follows Step 1 in the proof of Proposition 9 closely. Since D' contains D, the bank's optimized payoff must be as least as high

$$W_B\left(h^e_{D'}(\lambda); \lambda\right) \geq W_B\left(h^e_D(\lambda); \lambda\right) \quad \text{for all } \lambda \in \Lambda. \tag{38}$$

In all states $\lambda > \lambda^e_D$, the bank is bailed out and sets $h^e_D(\lambda)$ to either h_0 or h_1. Since the additions in moving to D' are all larger than both h_0 and h_1, Proposition 4 shows that the bailout received by the bank in these states cannot increase,

$$b\Big(h^e_{D'}(\lambda), y\left(h^e_{D'}(\lambda)\right)\Big) \leq b\Big(h^e_D(\lambda), y\left(h^e_D(\lambda)\right)\Big) \text{ for all } \lambda < \lambda^e_D. \tag{39}$$

For $\lambda \leq \lambda^e_D$, the bank's choice of h may either increase or decrease when we move to D'. However, since the bank is not bailed out under policy D, it must also not be bailed out under its optimal choice from D'. To see why, suppose it were bailed out under D'. Then $h^e_{D'}(\lambda)$ must equal either h_0 or h_1. But both of these options were available under D as well, contradicting the fact that the bank did not choose them and receive a bailout under policy D. We therefore have

$$b\Big(h^e_{D'}(\lambda), y\left(h^e_{D'}(\lambda)\right)\Big) = b\Big(h^e_D(\lambda), y\left(h^e_D(\lambda)\right)\Big) = 0 \text{ for all } \lambda \leq \lambda^e_D. \tag{40}$$

Combining equations (38) – (40) with the definition of $\mathcal{W}$ in equation (18) shows that we have $\mathcal{W}(D') \geq \mathcal{W}(D)$, that is, moving to delegation set D' weakly increases the regulator's payoff.

Next, we show that moving from D' to $\hat{D}$ in equation (37) also weakly increases the regulator's payoff. Note that $\hat{D}$ contains D' by construction, so we have the usual result that the bank's optimized payoff cannot decrease

$$W_B\left(h^e_{\hat{D}}(\lambda); \lambda\right) \geq W_B\left(h^e_{D'}(\lambda); \lambda\right) \quad \text{for all } \lambda \in \Lambda. \tag{41}$$

All that remains is to show that the bailout payment to the bank does not increase for any λ. Moving from D' to $\hat{D}$ adds choices of h that will cause a run if chosen when the bank is in the bailout region. For $\lambda > \lambda^e_{D'}$, any $h \in (h_0, \underline{h})$ is strictly inferior to choosing h_0. If the bank is going to suffer a run, it would prefer to set the smallest bail-in possible. Since h_0 was also available under D' and was not chosen, it must not be optimal under $\hat{D}$ either and the bank's optimal choice remains unchanged,

$$h^e_{\hat{D}}(\lambda) = h^e_{D'}(\lambda) \quad \text{for all } \lambda > \lambda^e_{D'}. \tag{42}$$

When $\lambda \leq \lambda^e_{D'}$, the bank is not bailed out under D'. In this case, the bank must not be bailed out following its optimal choice under $\hat{D}$ either. To see why, suppose it were bailed out under $\hat{D}$. Then its optimal choice $h^e_{\hat{D}}$ must be either h_0 or h_1. But both of these options were available under policy D', contradicting the fact that $\lambda \leq \lambda^e_{D'}$. Therefore, we have

$$b\Big(h^e_{\hat{D}}(\lambda), y\left(h^e_{\hat{D}}(\lambda)\right)\Big) = b\Big(h^e_{D'}(\lambda), y\left(h^e_{D'}(\lambda)\right)\Big) = 0 \text{ for all } \lambda \geq \lambda^e_{D'}. \tag{43}$$

Equations (41) - (43) imply we have

$$\mathcal{W}(\hat{D}) \geq \mathcal{W}(D') \geq \mathcal{W}(D),$$

as desired. Together, steps (i) and (ii) show that the optimal delegation set must have the form in equation (37). As discussed in the main text, we restrict the regulator to choose a closed set to ensure the bank's optimization problem has a solution in all states. If $h_1 > \underline{h}$, the bank may want to choose the bail-in h closest to $\underline{h}$ in some states where it is not bailed out, but no such closest number exists in $\hat{D}$. To avoid this technical complication, we approximate the form in equation (37) by

$$D^*_\varepsilon = [h_0, \underline{h} - \varepsilon] \cup [\underline{h}, 1],$$

and state our results in terms of the limiting case where ε approaches zero.

Step 3: Show at least one of $h^*_0 > 0$ and $h^*_1 > \underline{h}$ holds with strict inequality.

We establish the final step be contradiction. Suppose both $h^*_0 = 0$ and $h^*_1 = \underline{h}$ held. Then, taking the limiting case where $\varepsilon \to 0$, D^* would be all of the unit interval, as studied in Section 5. We will show that increasing one or both of these lower bounds would raise welfare, contradicting the claim that $D^* = [0, 1]$ is optimal. We break the analysis into cases based on the public sector's marginal cost of funds.

Case (i): $\mu_1 < \mu < \mu_2$

In this case, Proposition 6 establishes that $h^e(\lambda) = \underline{h} > 0$ for all $\lambda > \lambda^e$ when $D = [0, 1]$. In other words, in states where the bank is bailed out, it will choose the smallest bail-in that prevents a run. Moreover, $\mu < \mu_2$ implies this preference is strict, meaning the regulator can increase h_1 slightly above $\underline{h}$ and the bank will still prefer choosing h_1 over setting $h = 0$ and experiencing a run in those states where it is bailed out. Within this neighborhood, and keeping h^*_0 fixed at zero, we can write the regulator's expected payoff as a function of $h_1 \geq \underline{h}$

as

$$\int_0^{\underline{h}-\varepsilon} W_R(\lambda;\lambda)\,dF(\lambda) + \int_{\underline{h}-\varepsilon}^{\hat{\lambda}} W_R(\underline{h}-\varepsilon;\lambda)\,dF(\lambda) + \int_{\hat{\lambda}}^{h_1} W_R(h_1;\lambda)\,dF(\lambda) \tag{44}$$
$$+\int_{h_1}^{\lambda_D^e} W_R(\lambda;\lambda)\,dF(\lambda) + \int_{\lambda_D^e}^{\bar{\lambda}} W_R(h_1;\lambda)\,dF(\lambda)$$

where $\hat{\lambda}$ is the state where the bank is indifferent between h_1 and $\underline{h}-\varepsilon$, assuming it is not bailed out in either case,

$$W_B\left(h_1;\hat{\lambda}\right) = W_B\left(\underline{h}-\varepsilon;\hat{\lambda}\right), \tag{45}$$

and λ_D^e depends on h_1 as shown in equation (33) above. The first four terms in equation (44) correspond to states where the bank is not bailed out. When λ is less than $\underline{h}-\varepsilon$, the bank chooses the efficient bail-in $h=\lambda$. For λ between $\underline{h}-\varepsilon$ and h_1, the efficient bail-in lies in the "hole" of the delegation set and the bank must either bail-in less ($\underline{h}-\varepsilon$) or more ($h_1$). Equation (45) defines the cutoff below which the bank prefers $\underline{h}-\varepsilon$ and above which it prefers h_1. Finally, when λ is larger than λ_D^e, the bank chooses h_1 and is bailed out.

Differentiating the objective function with respect to h_1 yields

$$\int_{\hat{\lambda}}^{h_1} \frac{dW_R}{dh}(h_1;\lambda)\,dF(\lambda) + \left[W_R(\lambda_D^e;\lambda_d^e) - W_R(h_1,\lambda_D^e)\right] f(\lambda_D^e)\frac{d\lambda_D^e}{dh_1} \tag{46}$$
$$+\int_{\lambda_D^e}^{\bar{\lambda}} \frac{dW_R}{dh}(h_1;\lambda)\,dF(\lambda).$$

The first term in equation (46) captures the cost of distorting the bail-in in those states where the bank is not bailed out, the efficient bail-in lies in the "hole" of the delegation set, and the bank ends up choosing h_1. This term is negative for all $h_1 > \underline{h}$. The second term captures the change in the set of states where the bank is bailed out. The term in square brackets is positive when h_1 is close to $\underline{h}$. Since λ_D^e is increasing in h_1, this second term is strictly positive. The third term captures the effect of increasing the bail-in above $\underline{h}$ in states where the bank is bailed out. This term is also strictly positive when h_1 is close to $\underline{h}$. Note that no $d\hat{\lambda}/dh_1$ term appears in the derivative because the payoff function is continuous at $\hat{\lambda}$.

Evaluating this derivative at $h_1 = \underline{h}$ and taking the limit as $\varepsilon \to 0$, we have $\hat{\lambda} \to \underline{h}$ and the first term in equation (46) becomes zero. Since the other two terms remain strictly positive, the derivative is strictly positive at $h_1 = \underline{h}$. If $h_0^* = 0$, therefore, the optimal value of h_1^* must be strictly positive.

Case (ii): $\mu > \mu_2$

In this case, Proposition 7 establishes $h^e(\lambda) = 0$ for all $\lambda > \lambda^e$ when $D = [0,1]$. In other words, in states where the bank is bailed out, it chooses no bail-in and investors run on the bank. Moreover, $\mu > \mu_2$ implies this preference is strict, meaning the the regulator can increase h_0 slightly above 0 and the bank will still choose the lowest possible bail-in and experience a run in those states where it is bailed out. Within this neighborhood, and keeping h_1^* fixed at $\underline{h}$, we can write the regulator's expected payoff as a function of $h_0 \geq 0$ as

$$z\, W_R\,(h_0; 0) + \int_0^{h_0} W_R\,(h_0; \lambda)\, f(\lambda) d\lambda + \int_{h_0}^{\lambda_D^e} W_R(\lambda; \lambda) f(\lambda) d\lambda \\ + \int_{\lambda_D^e}^{\bar{\lambda}} W_R\,(h_0; \lambda)\, f(\lambda) d\lambda$$

where f is the density function for $\lambda > 0$ and $z \geq 0$ is the probability of $\lambda = 0$. Note that this equation looks nearly identical to the objective in equation (34) in the proof of Proposition 9 above, only with h_0 replacing h_1. The difference between the two equations lies inside the W_R term for $\lambda > \lambda_D^e$, which now captures the fact that a run is occurring in these states. Despite this difference, the steps are identical to those following equation (34) and are omitted here. Following those steps shows that, when $h_1 = \underline{h}$, increasing h_0 above zero creates a first-order gain for the regulator in states where the bank is bailed out and has no first-order cost in states where the bank is sound. As a result, $h_0^* > 0$ must hold.

Case (iii): $\mu = \mu_2$

The final case is where the public sector's marginal cost of funds lies exactly on the boundary between the two previous cases. The analysis in Section 5.2 of the main text shows that, in this case, the bank is indifferent between setting $h = \underline{h} > 0$, which prevents a run, and setting $h = 0$, which provokes a run. We assume the bank chooses $h = \underline{h}$ in this situation, but increasing h_1^* even slightly above $\underline{h}$ would lead the bank to switch to $h = 0$ in states where it is bailed out. To increase the regulator's payoff in this case, therefore, we need to raise both h_0 and h_1 together in such a way that the bank continues to be willing to choose the higher of the two values.

Give some $h_1 \geq \underline{h}$, let $g(h_1)$ be the bail-in satisfying

$$\pi u\left((1 - h_1)c_1^*\right) + (1 - \pi) u\left((1 - \lambda^*)c_1^*\right) = \\ \pi u\big((1 - g(h_1))c_1^*\big) + (1 - \pi)\big[\pi u\left((1 - \lambda^*)c_1^*\right) + (1 - \pi) u\left((1 - \lambda^*)c_2^*\right)\big]$$

In other words, the bank is indifferent between setting h_1 with no run and setting $g(h_1)$ with a run. Using the form of the utility function in equation (1), we can solve this equation for

$$g\,(h_1) = 1 - \left((1 - h_1)^{1-\gamma} + (1 - \pi)\left(\frac{u((1 - \lambda^*)c_2^*)}{u((1 - \lambda^*)c_1^*)} - 1\right)\right)^{\frac{1}{1-\gamma}}. \tag{47}$$

When $\mu = \mu_2$, we have $g(\underline{h}) = 0$. (This is effectively the definition of μ_2 from the proof of Proposition 6.) It is straightforward to show from equation (47) that $g(h_1)$ is strictly increasing and differentiable as h_1 increases above $\underline{h}$. When h_0 is set to $g(h_1)$, we can write the regulator's payoff as a function of h_1 as

$$z\,W_R\left(g(h_1);0\right)+\int_0^{g(h_1)}W_R\left(g(h_1);\lambda\right)f(\lambda)d\lambda+\int_{g(h_1)}^{\underline{h}-\varepsilon}W_R\left(\lambda;\lambda\right)f(\lambda)d\lambda$$
$$+\int_{\underline{h}-\varepsilon}^{\hat{\lambda}}W_R\left(\underline{h}-\varepsilon;\lambda\right)f(\lambda)d\lambda+\int_{\hat{\lambda}}^{h_1}W_R\left(h_1;\lambda\right)f(\lambda)d\lambda+\int_{h_1}^{\lambda_D^e}W_R\left(\lambda;\lambda\right)f(\lambda)d\lambda$$
$$+\int_{\lambda_D^e}^{\bar{\lambda}}W_R\left(h_1;\lambda\right)f(\lambda)d\lambda.$$

When the bank has zero loss or a small loss, it chooses the smallest allowable bail-in, $g(h_1)$. For λ between $g(h_1)$ and $\underline{h}-\varepsilon$, the bank is not bailed out and is able to choose the efficient bail-in, λ. For λ between $\underline{h}-\varepsilon$ and h_1 , the efficient bail-in lies in the hole of the delegation set and the bank will choose either $\underline{h}-\varepsilon$ or h_1. As in case (i) above, the cutoff state between these two choices, $\hat{\lambda}$, is given by equation (45). For λ between h_1 and λ_D^e, the bank is again able to choose the efficient bail-in, λ. Finally, for λ greater than λ_D^e, the bank chooses h_1 and prevents a run, as in case (i) above.

Differentiating this objective function with respect to h_1 yields

$$z\frac{dW_R}{dh}\left(g(h_1);0\right)g'(h_1)+\int_0^{g(h_1)}\frac{dW_R}{dh}\left(g(h_1);\lambda\right)g'(h_1)f(\lambda)d\lambda$$
$$+\int_{\hat{\lambda}}^{h_1}\frac{dW_R}{dh}\left(h_1;\lambda\right)f(\lambda)d\lambda+\Big[W_R\left(\lambda_D^e;\lambda_d^e\right)-W_R\left(h_1,\lambda_D^e\right)\Big]f(\lambda_D^e)\frac{d\lambda_D^e}{dh_1}$$
$$+\int_{\lambda_D^e}^{\bar{\lambda}}\frac{dW_R}{dh}\left(h_1;\lambda\right)f(\lambda)d\lambda.$$

We evaluate this derivative at $h_1=\underline{h}$ and take the limit as $\varepsilon\to 0$, which implies $\hat{\lambda}\to\underline{h}$ and, hence, the third term in the derivative is zero. In addition, $g(\underline{h})=0$ implies the second term is zero and the derivative reduces to

$$z\underbrace{\frac{dW_R}{dh}\left(0;0\right)}_{=0}g'(\underline{h})+\underbrace{\Big[W_R\left(\lambda_D^e;\lambda_D^e\right)-W_R\left(\underline{h},\lambda_D^e\right)\Big]}_{>0}f(\lambda_D^e)\underbrace{\frac{d\lambda_D^e}{dh_1}}_{>0}$$
$$+\int_{\lambda_D^e}^{\bar{\lambda}}\underbrace{\frac{dW_R}{dh}\left(h_1;\lambda\right)}_{>0}f(\lambda)d\lambda.\quad>\quad 0.$$

The first term measures the first-order cost of distorting the choice in states where the bank has no loss, which is shown to be zero in equation (36) above. The second term captures the benefit of shrinking the set of states where the bail is bailed out and is strictly positive. The final term captures the benefit of increasing the bail-in in states where the bank is bailed out, which is also positive. As a result, the derivative is strictly positive when evaluated at $h_1=\underline{h}$. The optimal policy must, therefore, have either $h_0^*>0$, $h_1^*>\underline{h}$, or both. □